Radio Frequency Engineering

Keywords: S-Parameter, Microwave device, Noise, Radio system.

Contents:

Radio Frequency network Analysis:

–EM Spectrum and Applications
–Electrical Length
–Physical Length
–Significance of Microwave Spectrum
–Application
–Scattering Matrix Parameters and Properties
–Insertion Loss, Return loss
–Transmission matrix (ABCD)
–Signal Flow Graph.

Radio Frequency Engineering

Keywords: S-Parameter, Microwave device, Noise, Radio system.

Contents:

Waveguide Based Devices:

Rectangular Waveguide Cavity

- –Cavity Resonator
- –Resonant Frequency and Quality Factor
- –Directional Coupler
- –Power Dividers and
- –IntroductionTee"s

Ferrites –Ferrite based Isolator and Circulator.

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Radio Frequency Systems:

Noise in RF Systems

- –Dynamic Range
- –Noise Equivalent Temperature
- –Noise Figure
- –Noise Figure of Cascaded System

Antenna Parameters

- –Gain
- –Directivity
- –Efficiency
- –Bandwidth
- –Beam width
- –Polarization

Dipole, Loops, Horn Antenna, Parabolic Dish

Frills Formula

Radio links

Introduction:

Microwaves are electromagnetic waves with a frequency:

300 MHz – 300 GHz (1 MHz = 10^6 Hz and 1 GHz = 10^9 Hz) or wavelengths in air ranging from 100 cm -to- 1 mm.

The word Microwave means very short wave, which is the shortest wavelength region of the radio spectrum and a part of the electromagnetic spectrum.

Microwave is an electromagnetic radiation of short wavelength.

Microwaves are easily attenuated within short distances.

They are not reflected by ionosphere

Transmission line techniques must be applied to short conductors

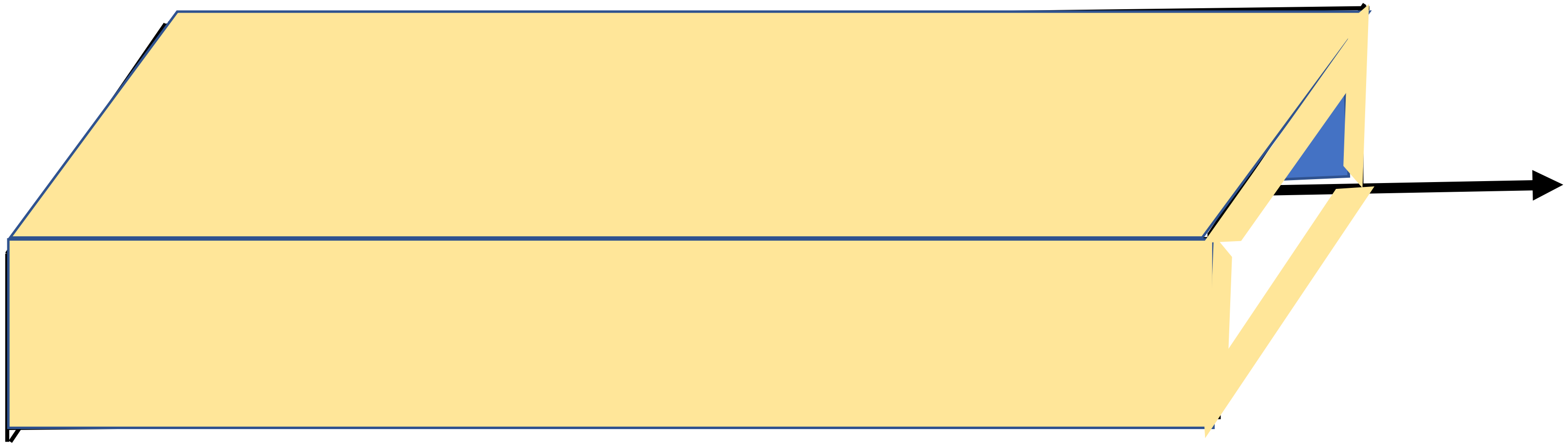
Propagations of this waves happens in such a way that direction of propagation, Electric field and Magnetic fields always remains perpendicular to each other.

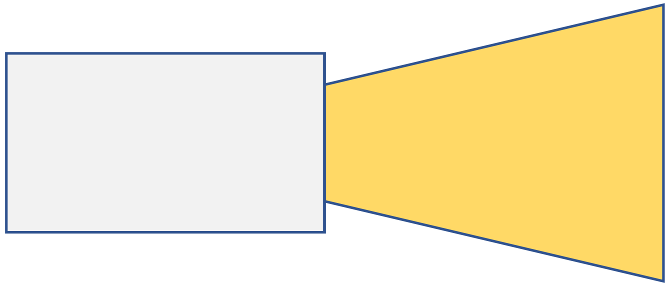
Microwaves frequencies characteristics are very much similar to light.

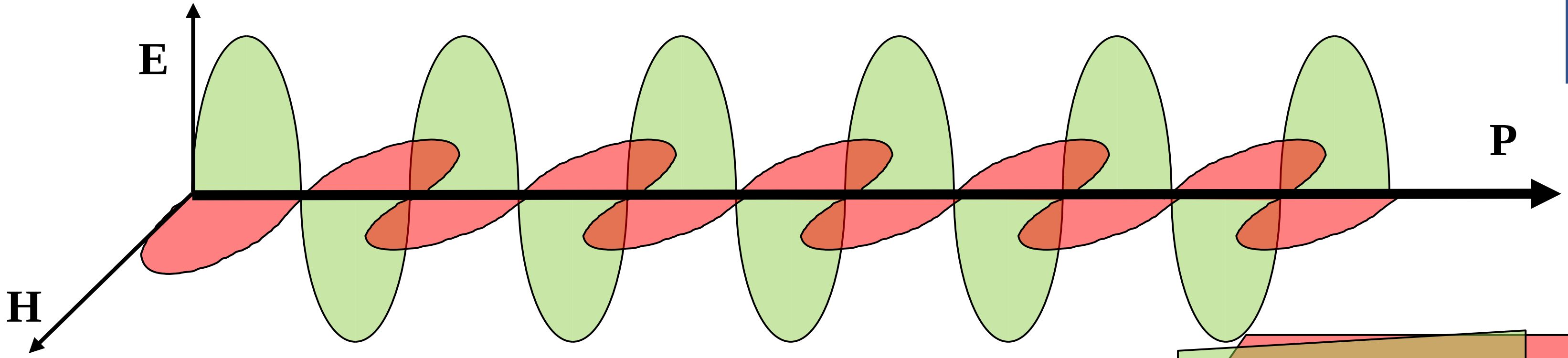
Stray reactance's are more important as frequency increases.

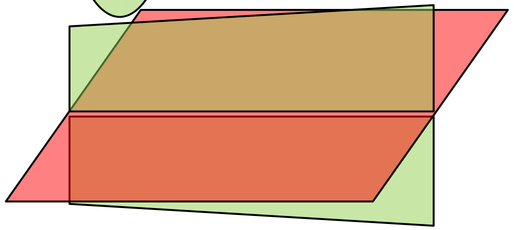
Solutions for the wave equations are obtained from the modified Maxwell's equations or Helmholtz's equations

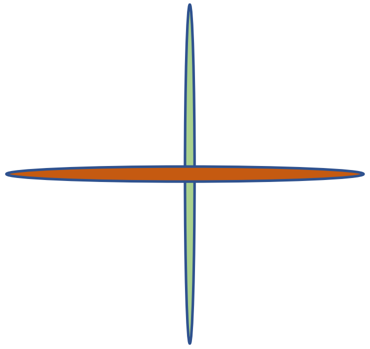
EM wave Propagation in Wave guides



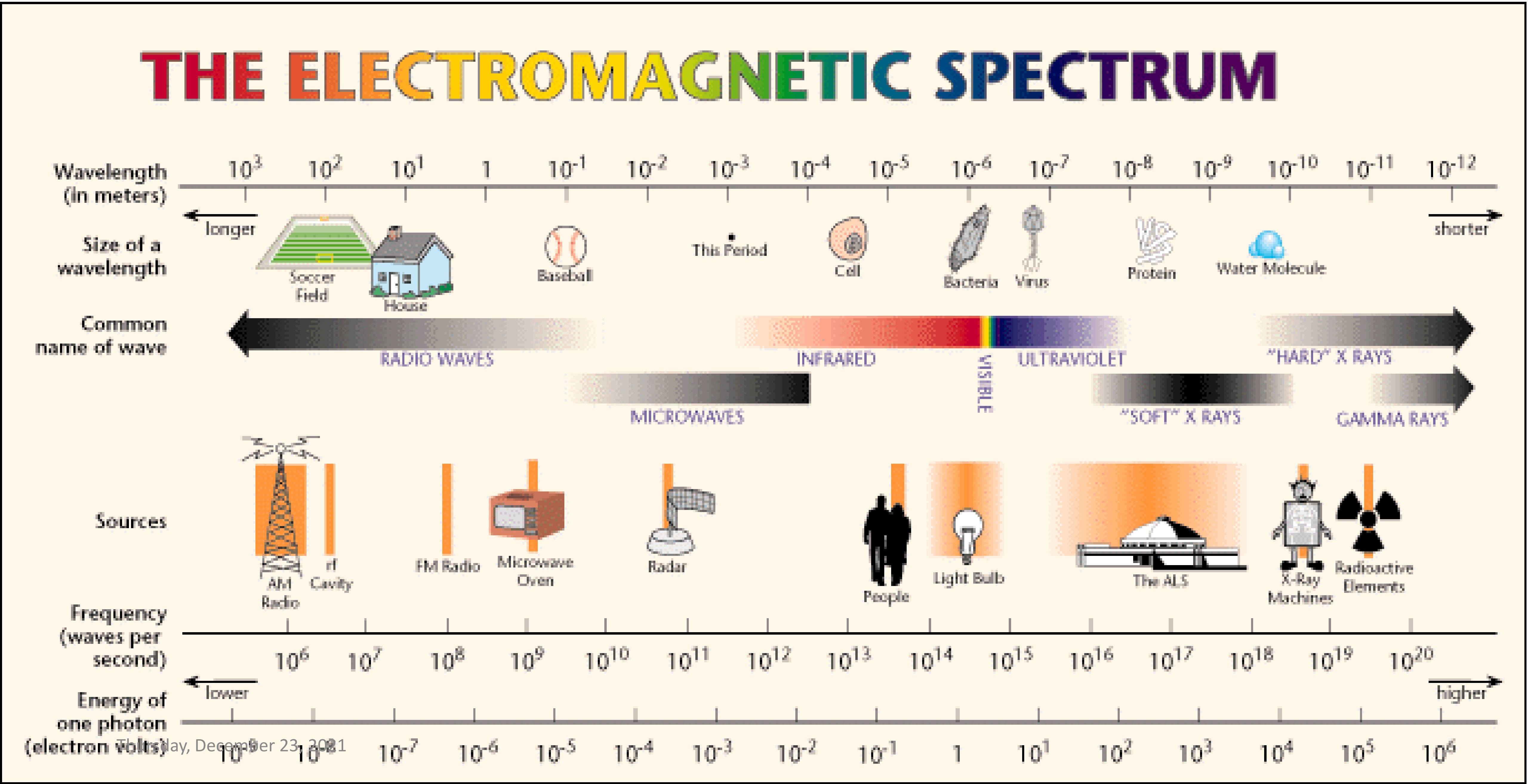








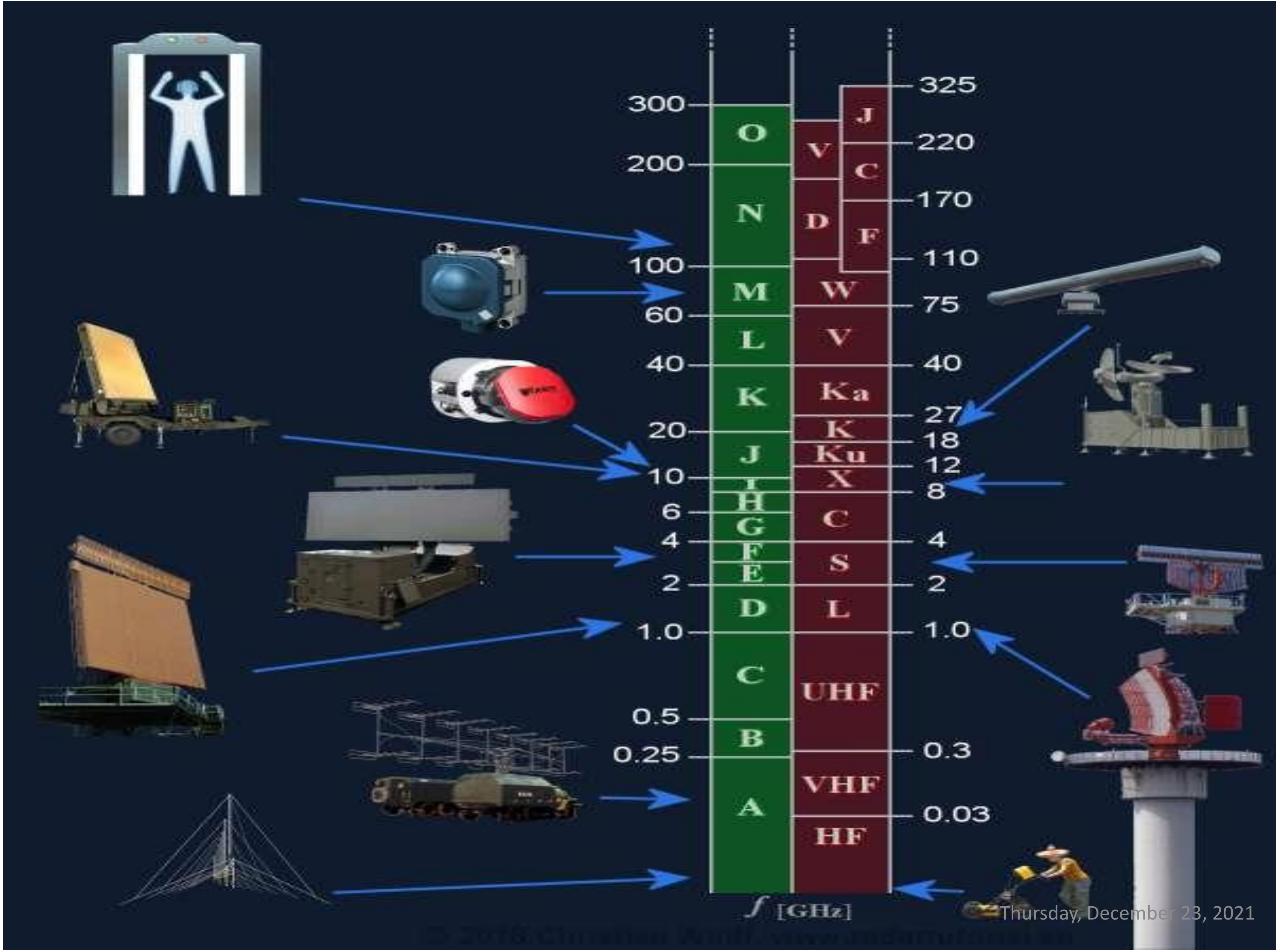
Electromagnetic Spectrum and Bands,



IEEE/Industry Standard band Designation

Band Designation		Frequency in GHz	Wavelength
UHF		0.3-3.0	0.999m-0.099m
L		1.1-1.7	0.273m-0.176m
LS		1.7-2.6	0.175m-0.115m
S		2.6-3.9	0.115m-0.768m
C		3.9-8.0	0.768m-0.037m
X		8.0-12.5	0.037m-0.024m
Ku		12.5-18.0	0.024m-0.017m
K		18-26	0.017m-0.011m
Ka		26-40	0.011m-0.007m
Q	Millimeter band	***33-50***	***0.009m-0.006m***
U		***40-60***	***0.007m-0.005m***
M		***50-75***	***0.006m-0.004m***
E		***60-90***	***0.005m-0.003m***
F		***90-140***	***0.003m-0.002m***
G		***140-220***	***0.002m-0.001m***
R		***220-300***	***0.001m-0.0009m***
Sub-millimetres		>300	<0.0009m

Microwave Bands for RADARS



Microwave Frequency Bands as per ITU Radio Regulation

Radio Waves are defined by Radio Regulations of the International telecommunication Union.

The radio spectrum allocated for Microwave are UHF,SHF and EHF as mentioned below in the table:

Band Number	Symbol	Frequency Range	Corresponding Metric Subdivision	Abbreviations for the band
4	VLF	3 to 30 kHz	Myriametric waves	B. Mam
5	LF	30 to 300 kHz	Kilometric waves	B. km
6	MF	300 to 3000 kHz	Hectometric waves	B. hm
7	HF	3 to 30 MHz	Decametric waves	B. dam
8	VHF	30 to 300 MHz	Metric waves	B. m
9	UHF	300 to 3000 MHz	Decimetric waves	B. dm
10	SHF	3 to 30 GHz	Centimetric waves	B. cm
11	EHF	30 to 300 GHz	Milimetric waves	B. mm
12		300 to 3000 GHz	Decimilimetric waves	

Advantages with microwaves

1. Increased bandwidth availability:

It provides larger BWs
More no. of channels will be allocated
More information will be transformed
Interference is less

- ❖ **To conclude;**
- ❖ Speech BW is 0.3 to 3kHz
- ❖ Music BW is 10 to 15 kHz
- ❖ TV Signal or FM BW is 5 to 7MHz

2. Improved directive properties:

As frequency increases, directivity increases from
The desired level of gain can be achieved with smaller dimensional antennas.

$$G = kD = k\left[\frac{4\pi A_e}{\lambda^2}\right] = k\left[\frac{4\pi A_e}{c^2}.f^2\right]$$

3. Fading effect and reliability:

$$E = h\upsilon; \text{where } \upsilon \text{ is the frequency}$$

Fading effect is less due to the axial uniformity in the waveguide structures.
Due to the Line of Sight (LOS) propagation and high frequencies, there is less fading effect and hence microwave communication is more reliable.

4. Power requirements:

- Transmitter / receiver power requirements are pretty low at microwave frequencies.

$$P_{rad} = \mu\,\pi^2\left(\frac{dl}{\lambda}\right)^2 I_{rms}^2 = \mu\,\pi^2\left(\frac{dl}{c^2}\right) f^2 I_{rms}^2$$

Advantages with Radio Waves

- **Increased bandwidth availability:**

As we go higher in frequency, fractional bandwidth increases. For example, let's assume that we wish to transmit a number of 4 kHz wide voice signals through a wireless link. Further let's assume that we have two wireless systems to chose from, one operating at 500 MHz and the second at 4 GHz, each with a 10 % bandwidth around its center

$$\text{No. of channels} = \frac{\text{Operating Frequency X \% of BW per channel}}{\text{Channel BW}}$$

❖ In theory, the 500 MHz system could carry:

$$\text{No. of channels} = \frac{0.5\text{GHz X } 0.1}{4\text{kHz}} = 12,500$$

❖ In theory, the 4 GHz system could carry:

$$\text{No. of channels} = \frac{4\text{GHz X } 0.1}{4\text{kHz}} = 100,000$$

❖ **To conclude;**

If some one is not required to go with 100000 no. of channels, then increase the % of BW per channel which increases the channel BW or guard bandwidth and hence the co-channel interference is greatly reduced.

Advantages & Applications-a brief

- **Advantages:**
  - Increased BW:
    - More no of channels
    - More BW per channel
    - More information transfer with less interference
  - Improved Directional properties
    - Increases Gain and other parameters with smaller dimensions
  - Improved Noise and interference parameters
    - Attenuations and Absorption are not severe
  - Transmitter and receiver power requirements are pretty low

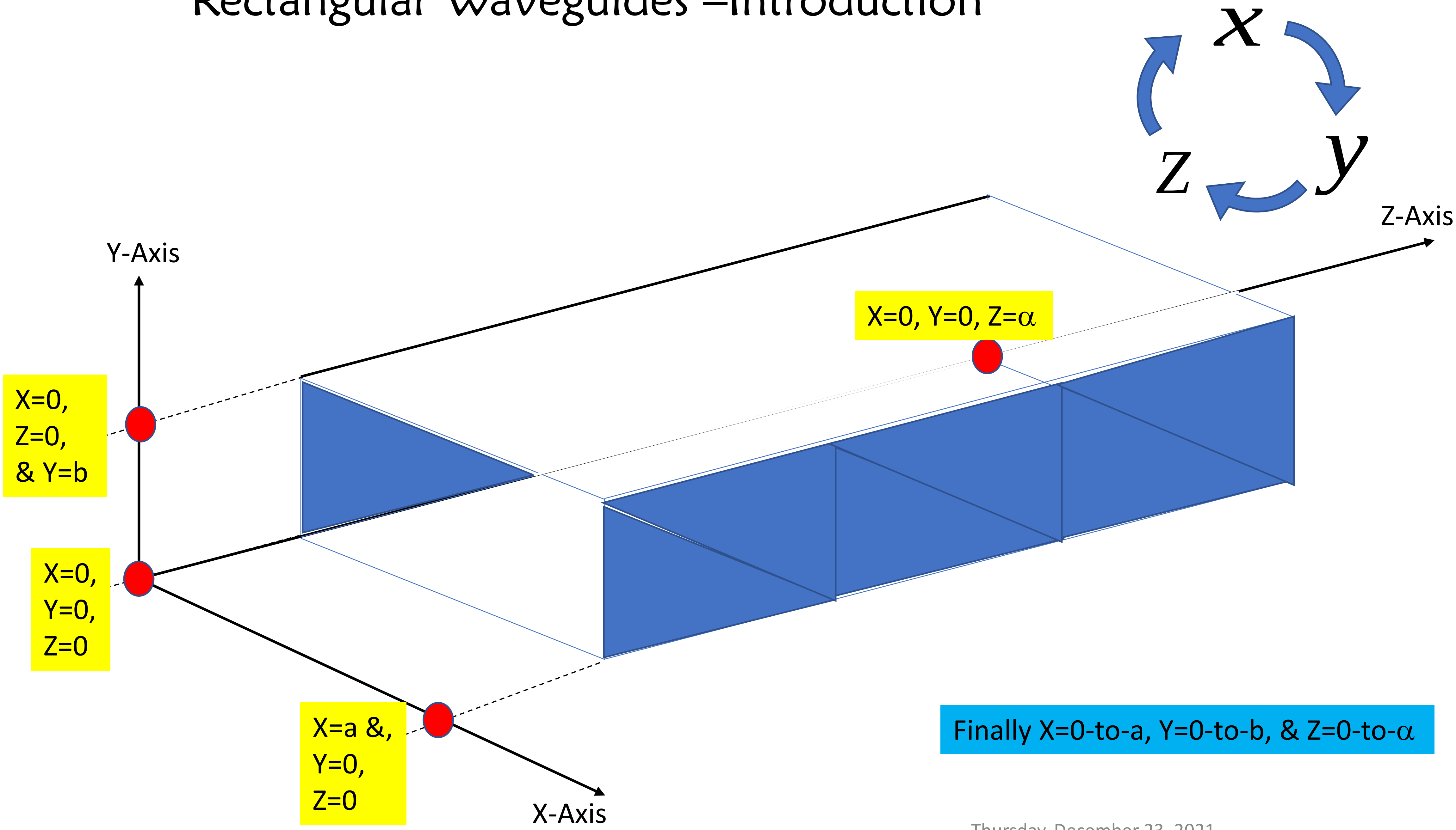
Advantages & Applications-a brief

- **Applications:**
  - •Wireless Communications (space, cellular phones, cordless phones, WLANs, Bluetooth, satellites etc.)
  - •Radar and Navigation (Airborne,vehicle, weather radars, GPS etc.)
  - •Remote sensing (Meteorology, mining, land surface, aviation and marine traffic etc.)
  - •RF Identification (Security, product tracking, animal tracking, toll collection etc.)
  - •Broadcasting (AM,FM radio, TV etc.)
  - •Heating (Baking, Food process, Ovens, Drying, Mining, rubber industry)
  - • Bio-medical application(Diagnostics)
  - •Jitter free switches
  - •Remote sensing applications
  - •Electronic WARFAREs ,ECMs….

Commercial and industrial applications

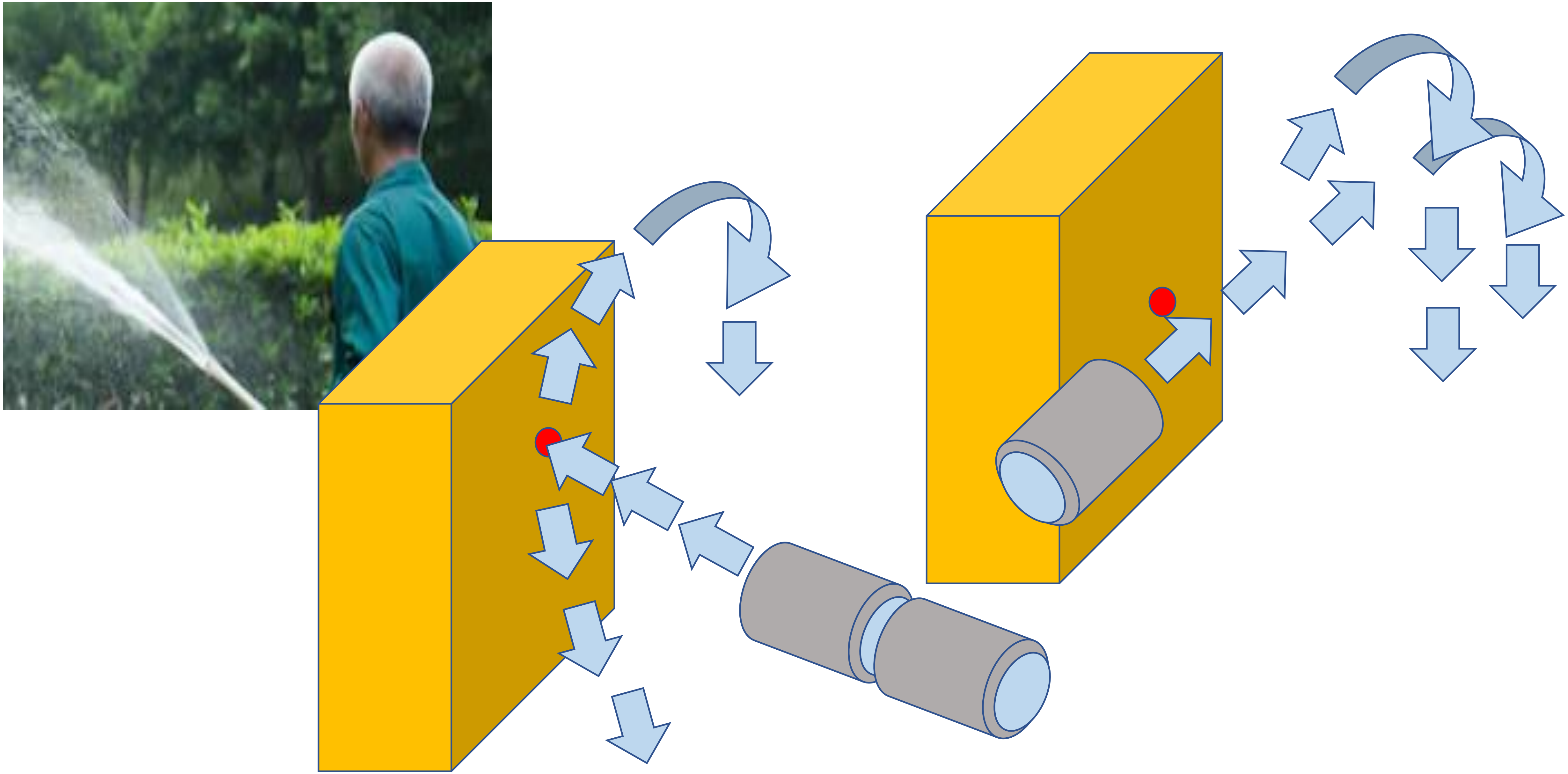
- Microwave oven
- Food process industry – Precooling / cooking, pasteurization / sterility, hat frozen / refrigerated precooled meats, roasting of food grains / beans.
- Rubber industry / plastics / chemical / forest product industries
- Mining / public works, breaking rocks, tunnel boring, drying / breaking up concrete, breaking up coal seams, curing of cement.
- Drying inks / drying textiles, drying / sterilizing grains, drying / sterilizing pharmaceuticals, leather, tobacco, power transmission.
- Biomedical Applications (diagnostic / therapeutic) – diathermy for localized superficial heating, deep electromagnetic heating for treatment of cancer, hyperthermia (local, regional or whole body for cancer therapy).
- Identifying objects or personnel by non – contact method.
- Light generated charge carriers in a microwave semiconductor make it possible to create a whole new world of microwave devices, fast jitter free switches, phase shifters, HF generators, etc.

Rectangular Waveguides –Introduction



Finally X=0-to-a, Y=0-to-b, & Z=0-to-α

Rectangular Waveguides –Introduction

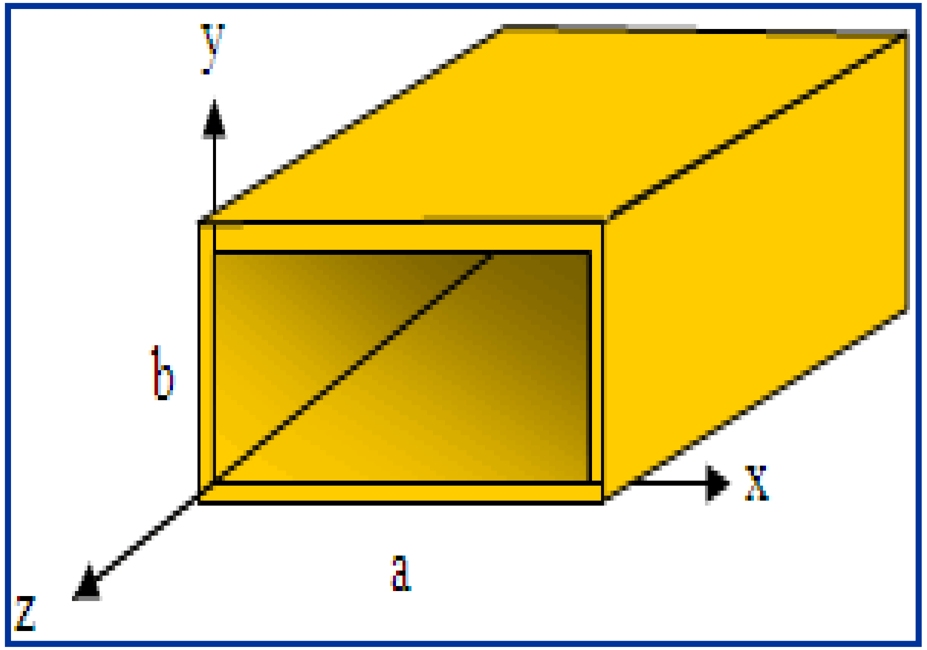


Thursday, December 23, 2021

Rectangular Waveguides –Field analysis of Transmission lines

- Any shape of cross section of a waveguide can support electromagnetic waves of which rectangular and circular waveguides have become more common.
- A waveguide having rectangular cross section is known as rectangular waveguide

- The size of the waveguide determines its operating frequency range.
- The frequency of operation is determined by the dimension ‘a’.
- This dimension is usually made equal to one – half the wavelength at the lowest frequency of operation, this frequency is known as the waveguide cutoff frequency.



- At the cutoff frequency and below, the waveguide will not transmit energy.
- At frequencies above the cutoff frequency, the waveguide will propagate energy.

Rectangular Waveguides –Field analysis of Transmission lines

A Hollow metallic tube of uniform cross section for transmitting electromagnetic waves by successive reflections from the inner walls of the tube is called ***waveguide*.**

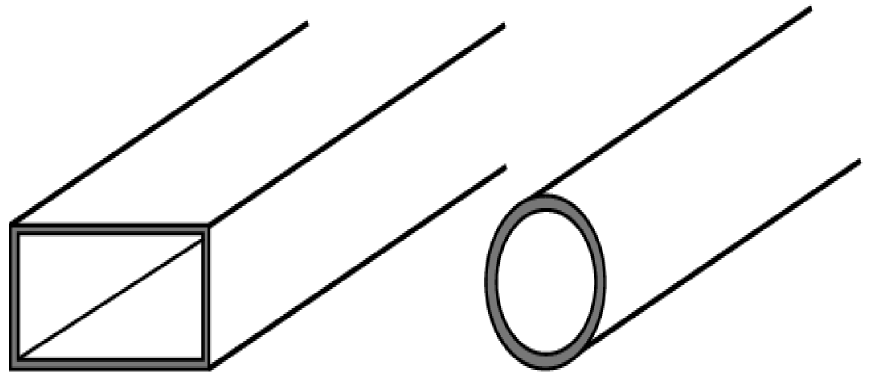
From the solutions of Maxwell's Equations, waves are classified into three types:

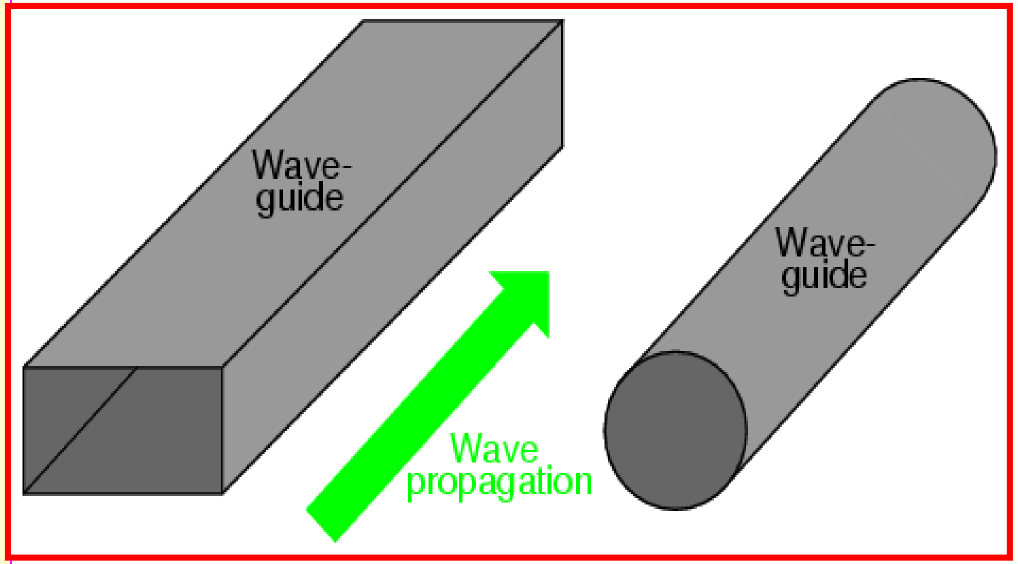
- Transverse Electro-Magnetic (TEM) wave
- Transverse Electric (TE) wave
- Transverse Magnetic (TM) wave

In **TEM mode**, both the electric and magnetic field components are transverse, or perpendicular to the direction of propagation.

In **TE mode**, the electric field component is transverse, or perpendicular to the direction of propagation.

In **TM mode**, the magnetic field component is transverse, or perpendicular to the direction of propagation.

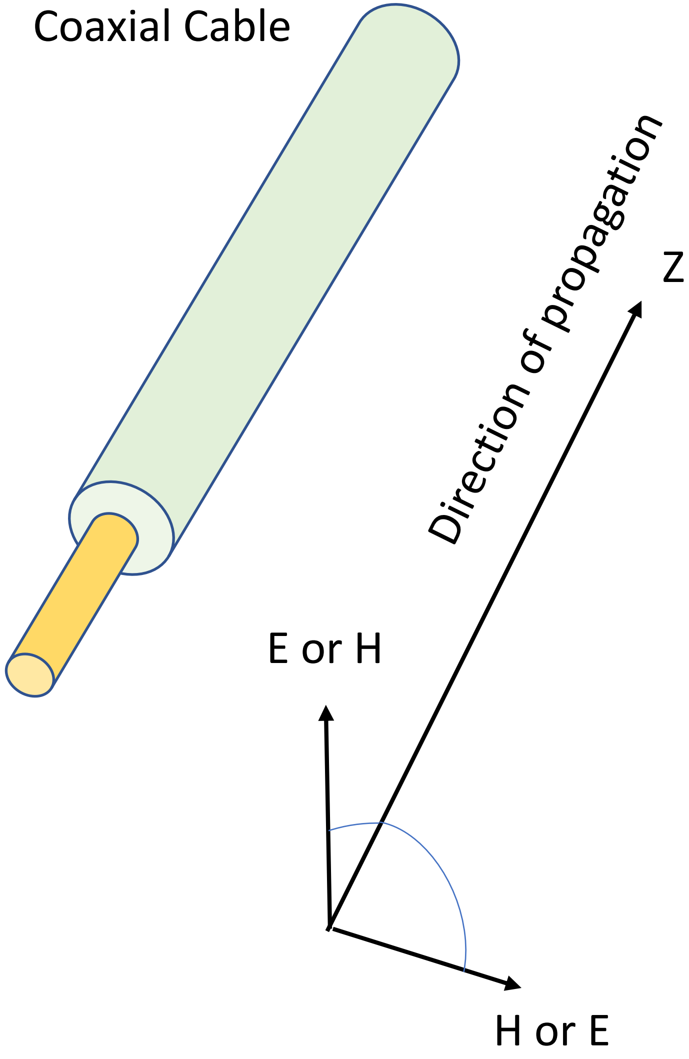




TEM mode Ez=Hz=0

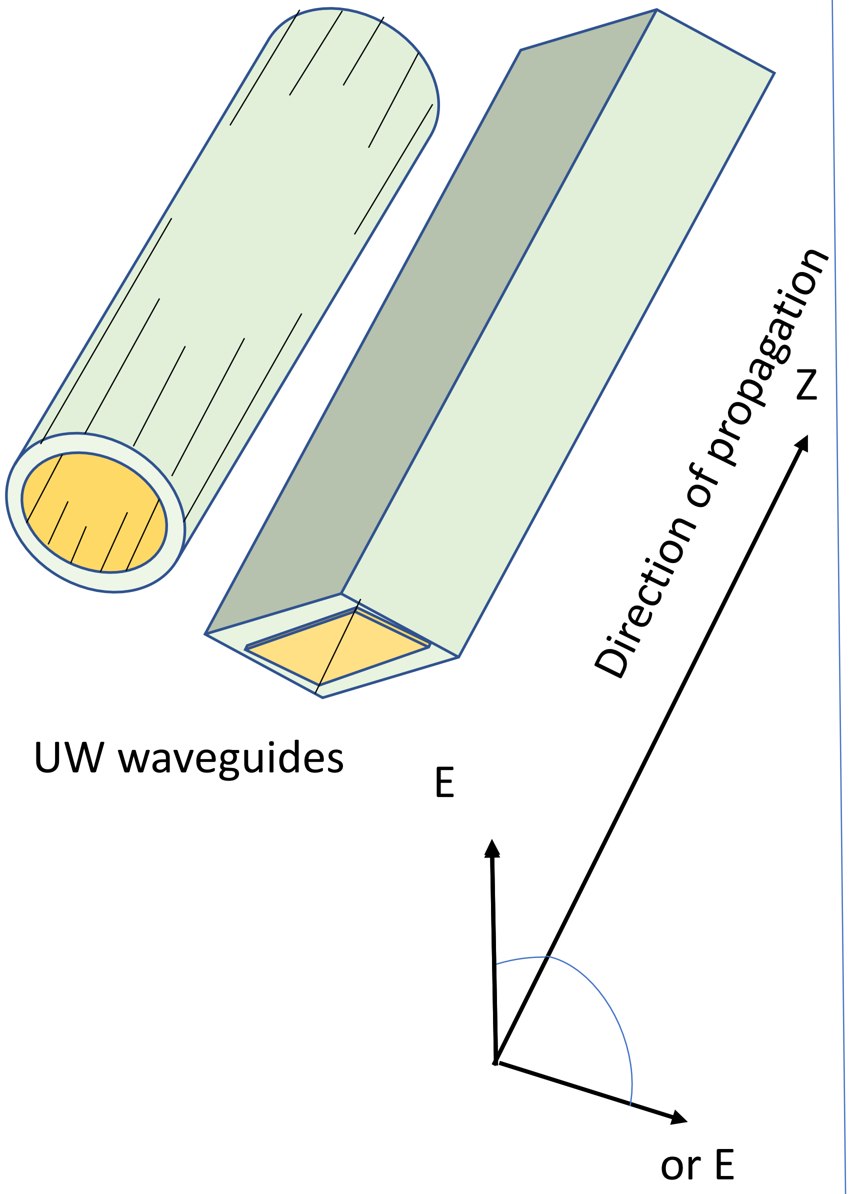
Conventional Transmission lines

Coaxial Cable



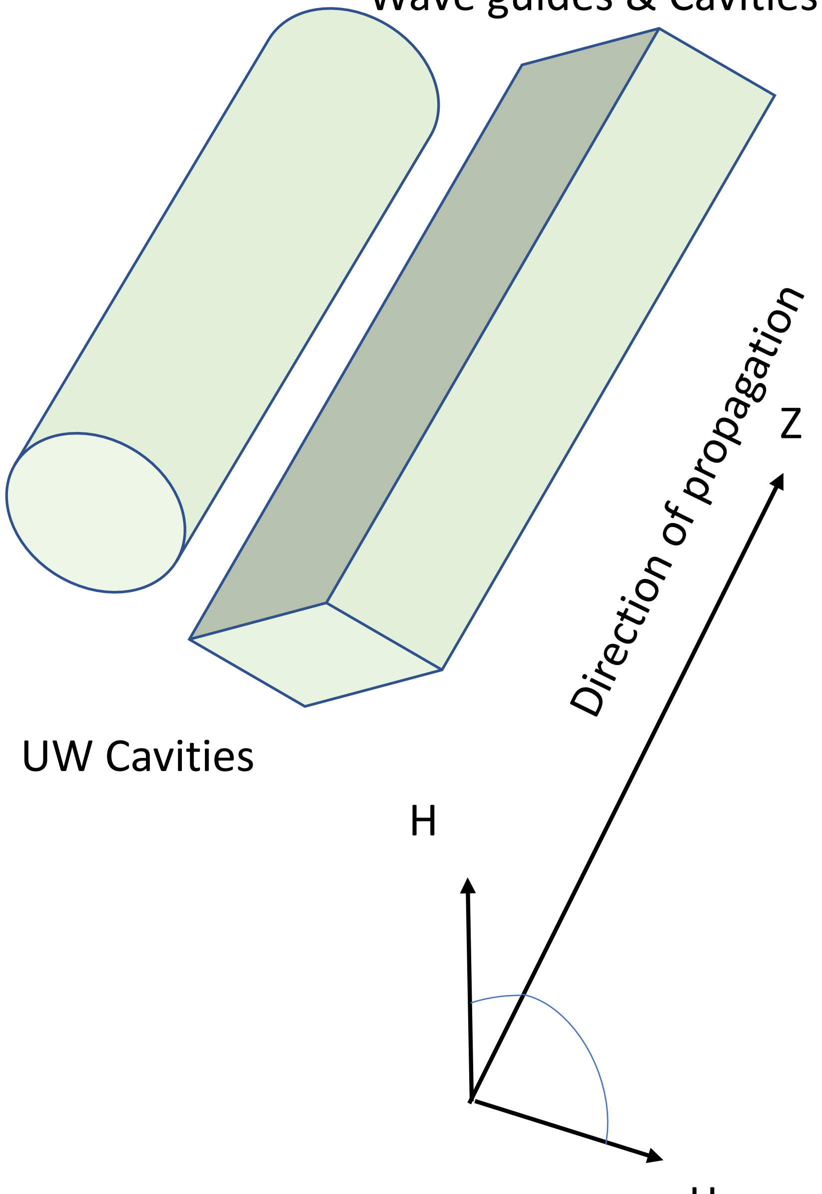
TE mode Ez=0 & Hz not=0

Wave guides & Cavities



TM mode Hz =0 & Ez not=0

Wave guides & Cavities



Representation of modes

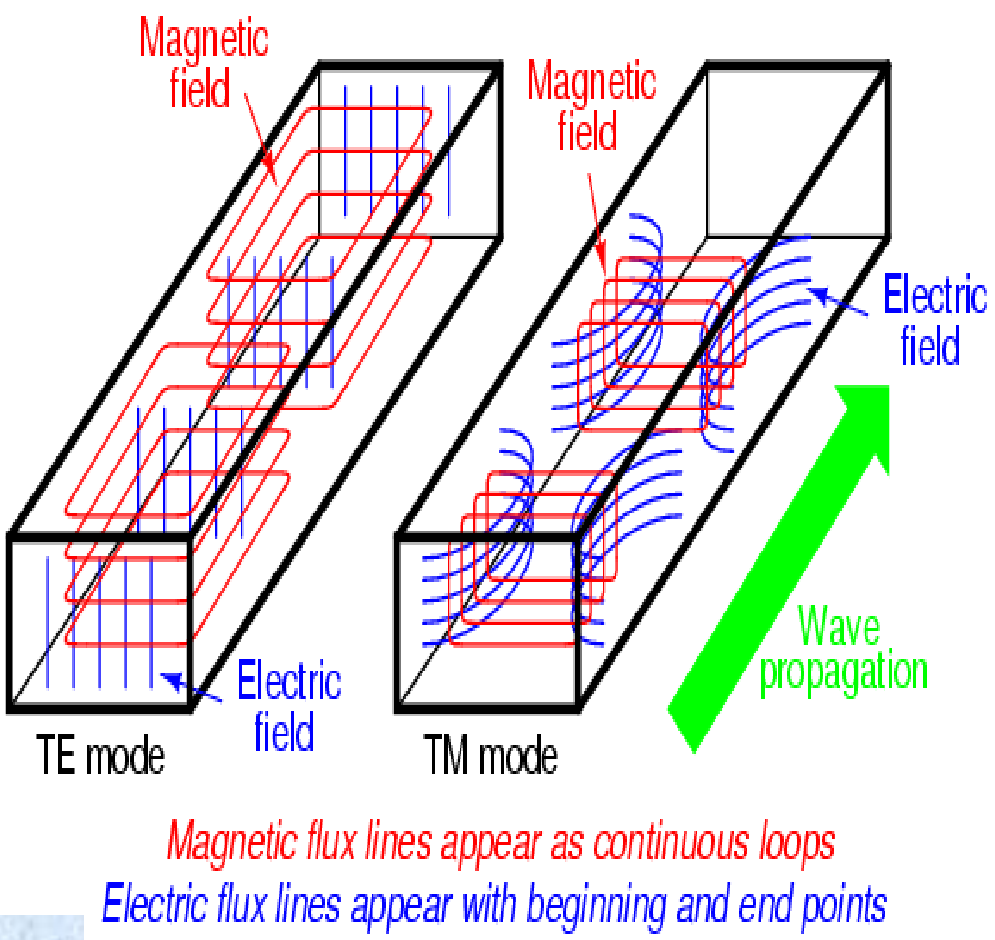
- The general symbolic of representation will be TE n, m or TM n, m

Where the first subscript n indicates the number of half wave variations of the electric field intensity along the a (wide) dimension of the waveguide

The second subscript m indicates the number of half wave variations of the electric field in the b (narrow) dimension of the guide. (with a=2b)

The TE 1, 0 mode has the longest operating wavelength and is designated as the dominant mode. It is the mode for the lowest frequency that can be propagated in a waveguide.

For a standard rectangular waveguide, the cutoff wavelength is given by,



$$\lambda_c = \frac{2}{\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2}}$$

Mode analysis-Classification of wave Solutions

Introduction:

Microwave transmission line are having axial uniformity- $\left(\frac{\partial ?}{\partial Z}=0\right)$

means that is cross-sectional shape and electrical properties do not vary w.rt length of the waveguide or along the axis, which is chosen as the z-axis.

Since the sources are not considered, the E and H components are solution of the homogenous vector Helmholtz Equations or Modified Maxwell's., ie…

$$\nabla^2 E + k_o^{\,2} E = 0;\ for\ TM\ waves$$

$$\nabla^2 H + k_o^{\,2} H = 0;\ for\ TE\ waves$$

Here, the "del" operator becomes, $\nabla = \nabla_t + \nabla_z = \nabla_t (e^{-j\beta z})$

Where ∇_t is transverse or perpendicular component wrt to z, the direction of propagation

$$\nabla_t = a_x . \frac{\partial}{\partial x} + a_y . \frac{\partial}{\partial y}$$

So ∇_z is tangential or parallel or axial component wrt to z.

Mode analysis-Classification of wave Solutions

So

$$E(x,y,z) = E_t(x,y,z) + E_z(x,y,z)$$

$$\Rightarrow E(x,y,z) = E(x,y) \quad E(z)$$

Parameter separable Property

$$\Rightarrow E(x,y,z) = E_z(x,y) = e(x,y)e^{-j\beta z}$$

and

$$H(x,y,z) = H_t(x,y,z) + H_z(x,y,z)$$

$$\Rightarrow H(x,y,z) = H(x,y) \quad H(z)$$

Parameter separable Property

$$\Rightarrow H(x,y,z) = H_z(x,y) = h(x,y)e^{-j\beta z}$$

Where E_t, H_t *are* transverse or perpendicular(x,y) components wrt to z.

and E_z, H_z are tangential or parallel or axial component wrt to z.

From this assumptions we will get reduced form of Maxwell's (or modified Maxwell's) equations, which are proved to be very useful in formulating the solutions for waveguide systems.

Mode analysis-Classification of wave Solutions

In order to determine the EM fields propagating within the waveguide, Modified Maxwell's equations should be solved subject to appropriate boundary conditions at the walls of the guide.

Such solutions give rise to a number of field configurations.

Each configuration is known as a mode. The following are the different modes possible in a waveguide system

Transverse Electro Magnetic (TEM) wave: Here both electric and magnetic fields are transverse components. (i.e.) $E_z = 0$ and $H_z = 0$

Transverse Electric (TE) wave: Here only the electric field is purely transverse to the direction of propagation and the magnetic field is not purely transverse. (i.e.) $E_z = 0$, $H_z \neq 0$. (Hence H-mode)

Transverse Magnetic (TM) wave: Here only magnetic field is transverse to the direction of propagation and the electric field component is not purely transverse. (i.e.) $E_z \neq 0$, $H_z = 0$. (Hence E-mode)

Procedure to Analyze Waveguides

- Solve reduced Helmholtz Equations ie..

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k_c^2\right)h_z=0 \;\; ; \text{for TE mode or H- mode}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k_c^2\right)e_z=0 \;\; ; \text{for TM mode or E - mode}$$

- The solution will contain several unknown constants(like A, Bs) wave numbers(k)
- Apply the boundary conditions to the appropriate field components to find out the unknown constants and k.
- Use the relation between known and unknown field components to find out the transverse fields (hz or ez).
- Find out all transverse field components from relations of wave impedance and propagation constants and other waveguide parameters $v_p, v_g, k_{c,nm}, f_{c,nm}, B_{nm}, Z_{nm}$

Basic equations used for;

Standard set of equations are..

$$H_z = \pm h.e^{\pm j\beta z} \pm h_z.e^{\pm j\beta z} = \pm h_z.e^{\pm j\beta z}$$

$$E_z = \pm e.e^{\pm j\beta z} \pm e_z.e^{\pm j\beta z} = \pm e_z.e^{\pm j\beta z}$$

$$H_x = \frac{j}{k_c^2}\left(\omega\varepsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x}\right)$$

$$H_y = \frac{-j}{k_c^2}\left(\omega\varepsilon\frac{\partial E_z}{\partial x} + \beta\frac{\partial H_z}{\partial y}\right)$$

$$E_x = \frac{-j}{k_c^2}\left(\beta\frac{\partial E_z}{\partial x} + \omega\mu\frac{\partial H_z}{\partial y}\right)$$

$$E_y = \frac{j}{k_c^2}\left(-\beta\frac{\partial E_z}{\partial y} + \omega\mu\frac{\partial H_z}{\partial x}\right)$$

k_c is the cut of wave number

$k_c^2 = k_x^2 + k_y^2$ or $k_c^2 = k_0^2 - \beta^2$

β is the propagation constant ($\beta = \frac{2\pi}{\lambda}$)

ω is the angular frequency($\omega = 2\pi f$)

μ is the permiability($\mu = \mu_r \mu_0$)

ε is the permittivity($\varepsilon = \varepsilon_r \varepsilon_0$)

TE wave or H- Mode analysis:

Transverse Electric (TE) wave: Here only the electric field is purely perpendicular to the direction of propagation and the magnetic field is not purely transverse.
(i.e.) E z = 0, Hz ≠ 0. (Hence H-mode)

- All the field components can be determined from the axial magnetic field hz by means of the following equation.

$$\nabla_t^{\,2} h_z + k_c^{\,2} h_z = 0; \quad \text{or} \quad \frac{\partial^2 h_z}{\partial x^2} + \frac{\partial^2 h_z}{\partial y^2} + k_c^{\,2} h_z = 0 \quad ---(1)$$

- Two imp' properties of wave guides solutions(by Helmholtz) are,
  - -Axial Uniformity and
  - -Parameter Separable

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial t} = 0$$

$$e(x, y, z) = e_t(x, y)e(z)$$

- Hence hz can be written as $h_z = f(x)g(y)$

- Equation 1 can be represented as

$$\frac{\partial^2 fg}{\partial x^2} + \frac{\partial^2 fg}{\partial y^2} + k_c^{\,2} fg = 0 \quad ---(2)$$

- Where f is a function of x - alone & g is a function of y - alone
- By considering the dependency of variables, equation 2 can be represented as,, ie.. independent variables can be take out from its derivative. Like..

$$g.\frac{\partial^2 f}{\partial x^2} + f.\frac{\partial^2 g}{\partial y^2} + k_c^{\,2} fg = 0 \quad ---(3)$$

- Dividing the equation 3 with fg and simplified for;

$$\Rightarrow \frac{1}{fg} g.\frac{\partial^2 f}{\partial x^2} + \frac{1}{fg} f.\frac{\partial^2 g}{\partial y^2} + \frac{1}{fg}.k_c^{\,2} fg = 0 ---(4)$$

- Whence

$$\Rightarrow \frac{1}{f}.\frac{\partial^2 f}{\partial x^2} + \frac{1}{g}.\frac{\partial^2 g}{\partial y^2} + \left(k_x^{\,2} + k_y^{\,2}\right) = 0 \quad ---(5)$$

•Equation 5 can be rearranged to

A partial differential equation with dependent variables converted into an equation of independent variables

$$0 - - - (6)$$

Function of x-alone

Function of y-alone

•So the equation 6 can be partitioned into two separate parts as given below

$$\frac{1}{f}\cdot\frac{\partial^2 f}{\partial x^2} + k_x^{\,2} = 0 \,(\text{or})\, \frac{\partial^2 f}{\partial x^2} + f\,k_x^{\,2} = 0 - - - (6A)$$

Function of x-alone

$$\frac{1}{g}\cdot\frac{\partial^2 g}{\partial y^2} + k_y^{\,2} = 0 \,(\text{or})\, \frac{\partial^2 g}{\partial y^2} + g\,k_y^{\,2} = 0 - - - (6B)$$

Function of y-alone

•Equation 6A with f and 6B with g are of the standard form $\frac{1}{Q}\cdot\frac{\partial^2 Q}{\partial y^2} = -n^2$

•whose solution is of the form

$$f = A_1\,\mathrm{Cos}(k_x x) + A_2\,\mathrm{Sin}(k_x x)$$

$$g = B_1\,\mathrm{Cos}(k_y y) + B_2\,\mathrm{Sin}(k_y y)$$

- Where A1, A2,B1,and B2 are arbitrary constants
- Kx and Ky are unknown wave number

These unknowns can be derived by applying the boundary conditions that hz satisfy and are defined by

$$\frac{\partial h_z}{\partial x} = 0 \text{ at x=0 and x=a} \quad \text{or} \quad \frac{df}{dx} = 0 \text{ at x=0 and x=a}$$

As f is a function of x-alone

$$\frac{\partial h_z}{\partial y} = 0 \text{ at y=0 and y=b} \quad \text{or} \quad \frac{dg}{dy} = 0 \text{ at y=0 and y=b}$$

As g is a function of y-alone

- So from the above equations; it can be

$$\frac{d}{dx}\left(A_1 \text{Cos}(k_x x) + A_2 \text{Sin}(k_x x)\right) = 0$$

$$\Rightarrow -k_x.A_1.\text{Sin}(k_x x) + k_x.A_2 \text{Cos}(k_x x) = 0$$

Substituting x=0 , gives $A_2 = 0$

at x=a, we have to take $\text{Sin}(k_x a)=0$

This means that $k_x a = n\pi$; *at* $n = 0,1,2,....$

or $k_x = n\frac{\pi}{a}$

- Whence $f = A_1 \text{Cos}(\frac{n\pi}{a} x)$ and $g = B_1 \text{Cos}(\frac{m\pi}{b} y)$

- But we know that hz=fg(from initial assumption)
- So we can write the solution for hz as

$$\therefore h_z = fg = A_1 \, Cos(\frac{n\pi}{a} x) B_1 \, Cos(\frac{m\pi}{b} y)$$

- Or it can also be modified like..

$$\therefore h_z = A_1 B_1 \, Cos(\frac{n\pi}{a} x) Cos(\frac{m\pi}{b} y) = A_{nm} \, Cos(\frac{n\pi}{a} x) Cos(\frac{m\pi}{b} y)$$

- Where Anm is an arbitrary constant for the nm th mode in a RWG
- But we know that the final component of magnetic field is given by

$$H_z = \pm h_z e^{\pm j\beta z}$$

Magnitude and phase of Hz

$$\therefore H_z = A_{nm} \, Cos(\frac{n\pi}{a} x) Cos(\frac{m\pi}{b} y) . e^{\pm j\beta z} \quad --A$$

and $E_z = 0$ from TE mode definition

- Equation A can be substituted in standard form to evaluate further required field components, like… Ex, Ey, Hx, and Hy

- As stated, equation A can be substituted, in the following set, to evaluate additional associated field components

Expressions in General

$$H_x = \pm \frac{j}{k_c^2}\left(\omega\varepsilon \frac{\partial E_z}{\partial y} - \beta \frac{\partial H_z}{\partial x} \right)$$

$$H_y = \pm \frac{j}{k_c^2}\left(\omega\varepsilon \frac{\partial E_z}{\partial x} + \beta \frac{\partial H_z}{\partial y} \right)$$

$$E_x = \pm \frac{j}{k_c^2}\left(\beta \frac{\partial E_z}{\partial x} + \omega\mu \frac{\partial H_z}{\partial y} \right)$$

$$E_y = \pm \frac{j}{k_c^2}\left(-\beta \frac{\partial E_z}{\partial y} + \omega\mu \frac{\partial H_z}{\partial x} \right)$$

Expressions for TE when Ez=0 ;

$$\Rightarrow H_x = \pm \frac{j}{k_c^2}\left(-\beta \frac{\partial H_z}{\partial x} \right) = \mp \frac{j\beta}{k_c^2} \cdot \frac{\partial H_z}{\partial x}$$

$$\Rightarrow H_y = \pm \frac{j}{k_c^2}\left(\beta \frac{\partial H_z}{\partial y} \right) = \frac{\pm j\beta}{k_c^2} \cdot \frac{\partial H_z}{\partial y}$$

$$\Rightarrow E_x = \pm \frac{j}{k_c^2}\left(\omega\mu \frac{\partial H_z}{\partial y} \right) = \pm \frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial y}$$

$$\Rightarrow E_y = \pm \frac{j}{k_c^2}\left(\omega\mu \frac{\partial H_z}{\partial x} \right) = \pm \frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial x}$$

Simplification

$$\because H_x = \mp \frac{j\beta}{k_c^2} \cdot \frac{\partial H_z}{\partial x}$$

$$\Rightarrow H_x = \mp \frac{j\beta}{k_c^2} \cdot \frac{\partial}{\partial x}\left(A_{nm}\, \mathrm{Cos}(\frac{n\pi}{a}x)\, \mathrm{Cos}(\frac{m\pi}{b}y).e^{\pm j\beta z} \right)$$

$$\Rightarrow H_x = \mp A_{nm} \frac{j\beta}{k_c^2}.\mathrm{Cos}(\frac{m\pi}{b}y).e^{\pm j\beta z} \frac{\partial}{\partial x}\left(\mathrm{Cos}(\frac{n\pi}{a}x) \right)$$

$$\Rightarrow H_x = \mp A_{nm} \frac{j\beta}{k_c^2}.\mathrm{Cos}(\frac{m\pi}{b}y).e^{\pm j\beta z} \left[-\mathrm{Sin}(\frac{n\pi}{a}x).\frac{n\pi}{a} \right]$$

$$\Rightarrow H_x = \pm A_{nm}.\frac{n\pi}{a}.\frac{j\beta}{k_c^2}.\mathrm{Sin}(\frac{n\pi}{a}x).\mathrm{Cos}(\frac{m\pi}{b}y).e^{\pm j\beta z}$$

$$\therefore H_x = \pm \frac{j\beta}{k_c^2} A_{nm}.\frac{n\pi}{a}.\mathrm{Sin}(\frac{n\pi}{a}x).\mathrm{Cos}(\frac{m\pi}{b}y).e^{\pm j\beta z} \quad --B$$

$$llly\ H_y = \pm \frac{j\beta}{k_c^2} A_{nm}.\frac{m\pi}{b}.\mathrm{Cos}(\frac{n\pi}{a}x).\mathrm{Sin}(\frac{m\pi}{b}y).e^{\pm j\beta z} \quad --C$$

$$\therefore H_z = A_{nm}\, \mathrm{Cos}(\frac{n\pi}{a}x)\, \mathrm{Cos}(\frac{m\pi}{b}y).e^{\pm j\beta z} \quad --A$$

$$\therefore E_x = \pm \frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial y}$$

$$\Rightarrow E_x = \pm \frac{j\omega\mu}{k_c^2} \frac{\partial}{\partial y}\left(A_{nm}\, \mathrm{Cos}(\frac{n\pi}{a}x)\, \mathrm{Cos}(\frac{m\pi}{b}y).e^{\pm j\beta z} \right)$$

$$\Rightarrow E_x = \pm A_{nm} \frac{j\omega\mu}{k_c^2} \mathrm{Cos}(\frac{n\pi}{a}x).e^{\pm j\beta z} \frac{\partial}{\partial y}\left(\mathrm{Cos}(\frac{m\pi}{b}y) \right)$$

$$\Rightarrow E_x = \pm A_{nm} \frac{j\omega\mu}{k_c^2} \mathrm{Cos}(\frac{n\pi}{a}x).e^{\pm j\beta z} \left[\mathrm{Sin}(\frac{m\pi}{b}y).\frac{m\pi}{b} \right]$$

$$\Rightarrow E_x = \pm A_{nm} \frac{m\pi}{b} \frac{j\omega\mu}{k_c^2} \mathrm{Cos}(\frac{n\pi}{a}x).\mathrm{Sin}(\frac{m\pi}{b}y).e^{\pm j\beta z}$$

$$\therefore E_x = \pm \frac{j\omega\mu}{k_c^2} A_{nm} \frac{m\pi}{b} \mathrm{Cos}(\frac{n\pi}{a}x).\mathrm{Sin}(\frac{m\pi}{b}y).e^{\pm j\beta z} \quad --D$$

$$llly\ E_y = \pm \frac{j\omega\mu}{k_c^2} A_{nm} \frac{n\pi}{a} \mathrm{Sin}(\frac{n\pi}{a}x)\, \mathrm{Cos}(\frac{m\pi}{b}y).e^{\pm j\beta z} \quad --E$$

and $E_z = 0 \quad --F$

Field expressions for RWGs in TE mode

$$\therefore H_x = \pm\frac{j\beta}{{k_c}^2}A_{nm}.\frac{n\pi}{a}.\mathrm{Sin}(\frac{n\pi}{a}x).\mathrm{Cos}(\frac{m\pi}{b}y).e^{\pm j\beta z}$$

$$H_y = \pm\frac{j\beta}{{k_c}^2}A_{nm}.\frac{m\pi}{b}.\mathrm{Cos}(\frac{n\pi}{a}x).\mathrm{Sin}(\frac{m\pi}{b}y).e^{\pm j\beta z}$$

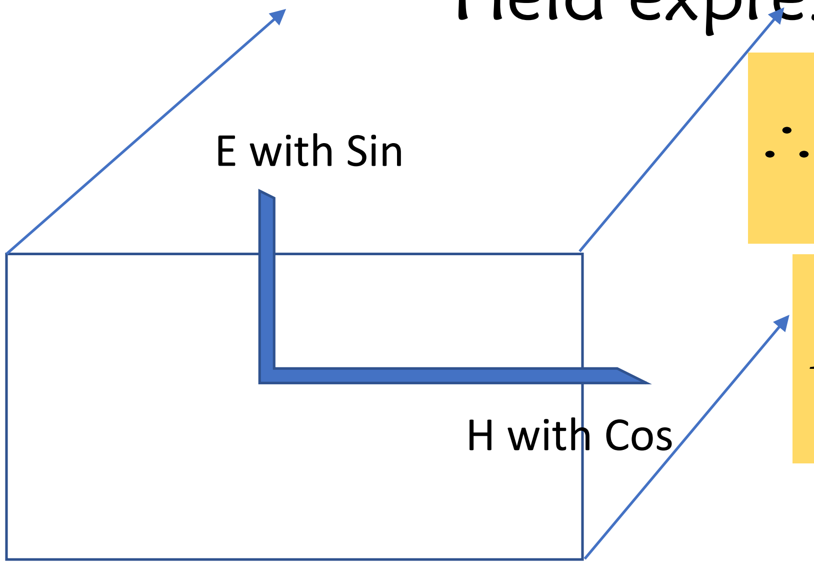
$$H_z = A_{nm}\,\mathrm{Cos}(\frac{n\pi}{a}x)\,\mathrm{Cos}(\frac{m\pi}{b}y).e^{\pm j\beta z}$$

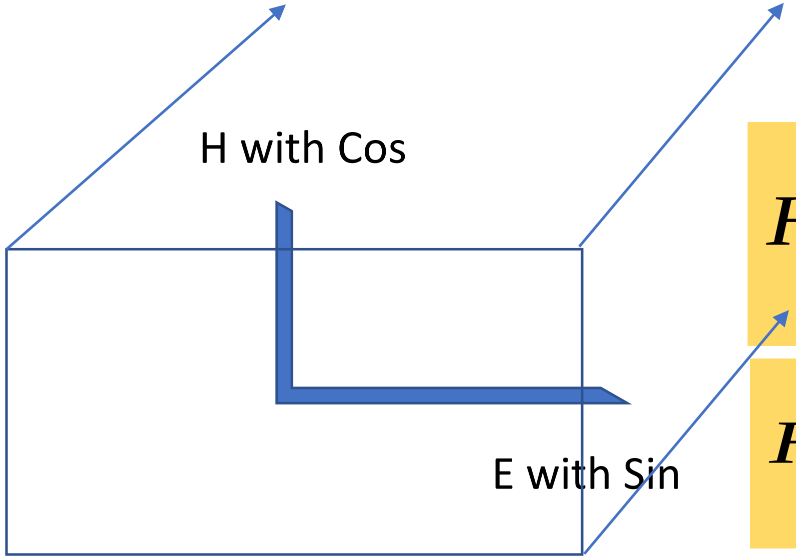
$$E_x = \pm\frac{j\omega\mu}{{k_c}^2}A_{nm}\,\frac{m\pi}{b}\mathrm{Cos}(\frac{n\pi}{a}x).\mathrm{Sin}(\frac{m\pi}{b}y).e^{\pm j\beta z}$$

$$E_y = \pm\frac{j\omega\mu}{{k_c}^2}A_{nm}\,\frac{n\pi}{a}\mathrm{Sin}(\frac{n\pi}{a}x)\,\mathrm{Cos}(\frac{m\pi}{b}y).e^{\pm j\beta z}$$

$$E_z = 0$$

Field expressions for RWGs in TM mode





$$\therefore E_x = \pm \frac{j\beta}{{k_c}^2} A_{nm} . \frac{n\pi}{a} . Cos(\frac{n\pi}{a} x). Sin(\frac{m\pi}{b} y). e^{\pm j\beta z}$$

$$E_y = \pm \frac{j\beta}{{k_c}^2} A_{nm} . \frac{m\pi}{b} . Sin(\frac{n\pi}{a} x). Cos(\frac{m\pi}{b} y). e^{\pm j\beta z}$$

$$E_z = A_{nm}\, Sin(\frac{n\pi}{a} x) Sin(\frac{m\pi}{b} y). e^{\pm j\beta z}$$

$$H_x = \pm \frac{j\omega\mu}{{k_c}^2} A_{nm} \frac{m\pi}{b} Sin(\frac{n\pi}{a} x). Cos(\frac{m\pi}{b} y). e^{\pm j\beta z}$$

$$H_y = \pm \frac{j\omega\mu}{{k_c}^2} A_{nm} \frac{n\pi}{a} Cos(\frac{n\pi}{a} x) Sin(\frac{m\pi}{b} y). e^{\pm j\beta z}$$

$$H_z = 0$$

So solutions with COS or SIN are valid solutions for both TE & TM wrt plane of reference

Characteristic Equation and Cut-off Frequencies

- The rectangular waveguide characteristic equation or characteristic impedance is given by

$$\eta_{nm} = Z_{nm} = \frac{E_x}{H_y} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_o\mu_r\varepsilon_o\varepsilon_r}} = \frac{1}{\sqrt{\mu_o\varepsilon_o}}\frac{1}{\sqrt{\mu_r\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

- A particular mode is only supported above its cutoff frequency and the cutoff frequency is given by

where $c = 3\times10^8$ m/s

$$f_{c_{nm}} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2} = \frac{c}{2\sqrt{\mu_r\varepsilon_r}}\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2} \text{ and } \lambda_{cnm} = \frac{c}{f_{cnm}}$$

- Propagation constant $\gamma = j\beta$ when $\alpha = 0$

- Phase velocity Vp is

$$\upsilon_p = \frac{c}{\sqrt{1-\left(f_c/f_0\right)^2}} = \frac{c}{\sqrt{1-\left(\lambda_0/\lambda_c\right)^2}}$$

- Group velocity Vg is

$$\upsilon_g = c\sqrt{1-\left(f_c/f_0\right)^2} = c\sqrt{1-\left(\lambda_0/\lambda_c\right)^2}$$

The relation between $\lambda_0 \lambda_c \lambda_g$ is given by

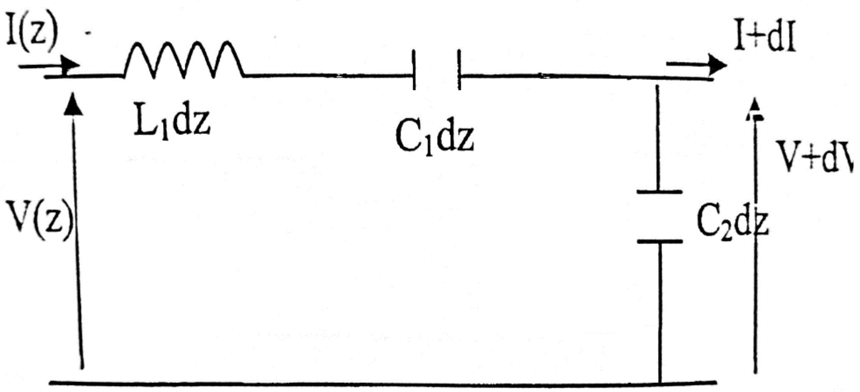
$$\frac{1}{\lambda_0^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$$

Cut off wave number k_c is defined as

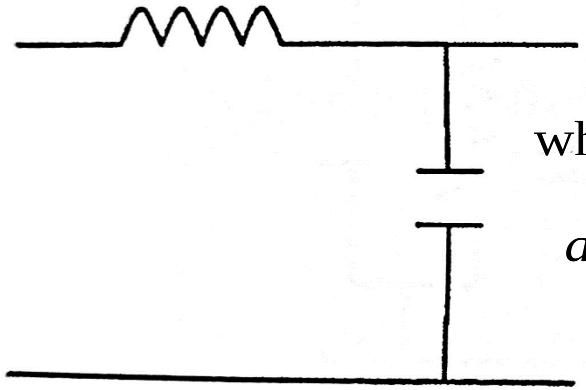
$$k_c = \sqrt{k_x^2 + k_y^2} = \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}$$

Filter Characteristics

- I was reading in a book how to derive the equivalent circuits for TE, TM, TEM modes of a generic ideal (without losses) waveguide. After some computations, I found this equivalent circuit for TE modes:
- It is written that for f > fc (cut - off frequency of the mode), this structure allows propagation because L1 dominates on C1, while for f < fc there is not propagation because C1 dominates on L1. It is shown in the following schemes:



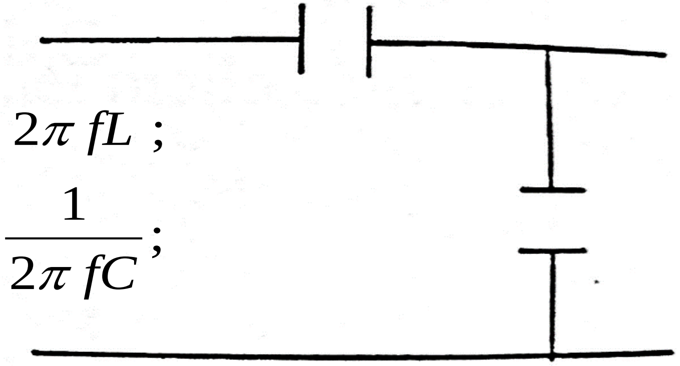
Equivalent circuit for TE modes



(a) $\omega > \omega_c$

where $X_L = 2\pi fL$;

and $X_C = \dfrac{1}{2\pi fC}$;



(b) $\omega < \omega_c$

- **First question:** why should the circuit at left allow propagation, and that at right not?

Filter Characteristics

- A waveguide filter is an electronic filter constructed with waveguide technology. Waveguides are hollow metal conduits inside which an **electromagnetic wave** may be transmitted.
- Filters are devices used to **allow signals** at some frequencies to pass (the passband), while others are **rejected** (the stopband).
- Filters are a basic component of electronic engineering designs and have numerous applications., viz. **selection** or **rejection** of signals and to **limit the noise**.
- Waveguide filters are most useful in the microwave band of frequencies, where they are a convenient size and have low loss.
- Examples of microwave filter use are found in **satellite communications, telephone networks, and television broadcasting.**
- A particular feature of waveguide filter design concerns the **mode of transmission.**
- In waveguide systems, **any number of modes are possible.** Where as in pairs of conducting wires and similar technologies have only **one mode of transmission.**

•Waveguide filters have much more in common with transmission line filters than lumped element filters; they do **not contain any discrete capacitors or inductors**.

•However, the waveguide design may frequently be equivalent (or approximately so) to a lumped element design.

•The design of waveguide filters frequently starts from a **lumped elements** design and then converts the elements of that design into waveguide components

•Another **peculiar feature** to waveguide filters is that there is a definite frequency, **the cutoff frequency**, below which **no transmission can take place**.

•The filter is consequently low-pass by design and may be considered a low-pass filter for all practical purposes if the **cutoff frequency is below any frequency of interest** to the application.

• The waveguide cutoff frequency is a function of **transmission mode**, so at a given frequency, the waveguide may be usable in some modes but not others. Likewise, the **guide wavelength(λ_g)** and **characteristic impedance (Z_0)** of the guide at a given frequency also depend on mode

Dominant and Degenerate Modes

•The dominant mode of propagation is the one with the lowest possible cut-off frequency.

•A typical guide may have a=2b.in which case:

$$\lambda_{cnm} = \frac{2}{\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2}} = \frac{2}{\sqrt{\frac{n^2}{a^2} + \frac{m^2}{b^2}}} = \frac{2}{\sqrt{\frac{n^2b^2 + m^2a^2}{a^2b^2}}} = \frac{1}{\frac{1}{ab}} \frac{2}{\sqrt{n^2b^2 + m^2a^2}} = \frac{2ab}{\sqrt{n^2b^2 + m^2a^2}}$$

$$\Rightarrow \lambda_{c10} = \frac{2.2b.b}{\sqrt{1^2b^2 + 0^2(2b)^2}} = \frac{4.b^2}{\sqrt{b^2}} = 4b = 2(2b) = 2a$$

$$\Rightarrow \lambda_{c01} = \frac{2.2b.b}{\sqrt{0^2b^2 + 1^2(2b)^2}} = \frac{4.b^2}{\sqrt{0^2b^2 + 1^2(2b)^2}} = \frac{4.b^2}{\sqrt{4b^2}} = \frac{4.b^2}{2b} = 2b = a$$

$$\Rightarrow \lambda_{c11} = \frac{2.2b.b}{\sqrt{1^2b^2 + 1^2(2b)^2}} = \frac{4b^2}{\sqrt{b^2 + 4b^2}} = \frac{4b^2}{\sqrt{5b^2}} = \frac{4b^2}{b\sqrt{5}} = \frac{4b}{\sqrt{5}} = \frac{2a}{\sqrt{5}}$$

•The corresponding frequency is $\Rightarrow f_{c10} = \frac{c}{2a}$ and $f_{c01} = \frac{c}{a}$ and hence $\frac{c}{2a} < f < \frac{c}{a}$

Dominant and Degenerate Modes

•There by with in the specified range only H10 mode will propagate and hence this (TE10) mode is called the DOMINANT mode, and hence it is desired to find out it's associated field components. Such as

$$H_{z10} = A_{10}\,\text{Cos}(\frac{\pi}{a}x).e^{\pm j\beta z}$$

$$\therefore H_{x10} = \pm\frac{j\beta}{k_c^{\,2}}A_{10}.\frac{\pi}{a}.\text{Sin}(\frac{\pi}{a}x).e^{\pm j\beta z}$$

$$E_{y10} = \pm\frac{j\omega\mu}{k_c^{\,2}}A_{10}\frac{\pi}{a}\text{Sin}(\frac{\pi}{a}x).e^{\pm j\beta z}$$

$$H_{y10} = \pm\frac{j\beta}{k_c^{\,2}}A_{10}.\frac{0\pi}{b}.\text{Cos}(\frac{n\pi}{a}x).\text{Sin}(\frac{0\pi}{b}y).e^{\pm j\beta z} = 0$$

$$E_{x10} = \pm\frac{j\omega\mu}{k_c^{\,2}}A_{10}\frac{m\pi}{b}\text{Cos}(\frac{\pi}{a}x).\text{Sin}(\frac{0\pi}{b}y).e^{\pm j\beta z} = 0$$

$$E_z = 0$$

$$\text{where } k_c{}^2{}_{nm} = k_x{}^2 + k_y{}^2 = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2$$

$$\Rightarrow k_c{}^2{}_{10} = k_x{}^2 + k_y{}^2 = \left(\frac{1.\pi}{a}\right)^2 + \left(\frac{0.\pi}{b}\right)^2 = \left(\frac{\pi}{a}\right)^2$$

$$\text{and } \beta_{nm}{}^2 = k_0{}^2 - k_c{}^2{}_{nm} \text{ or } \beta_{nm} = \left(k_0{}^2 - k_c{}^2{}_{nm}\right)^{\frac{1}{2}}$$

$$\Rightarrow \beta_{10}{}^2 = k_0{}^2 - k_c{}^2{}_{10} \text{ or } \beta_{10} = \left(k_0{}^2 - \left(\frac{\pi}{a}\right)^2\right)^{\frac{1}{2}}$$

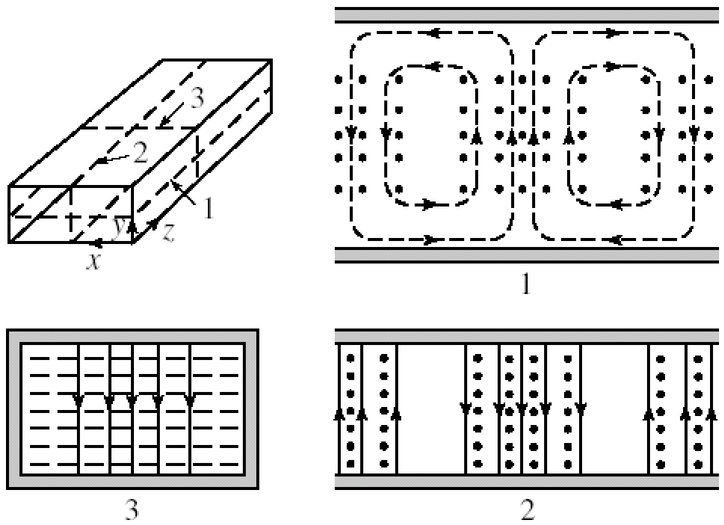
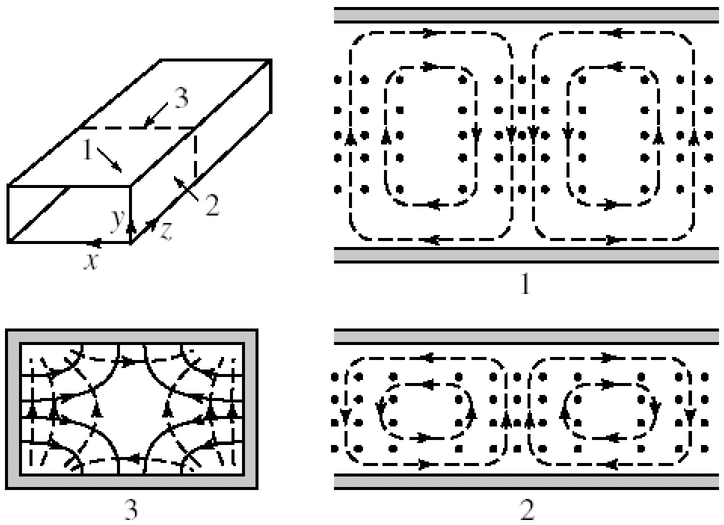
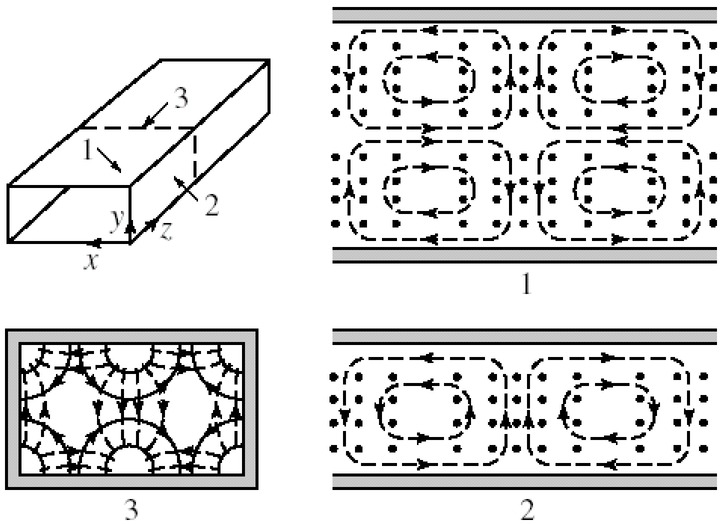
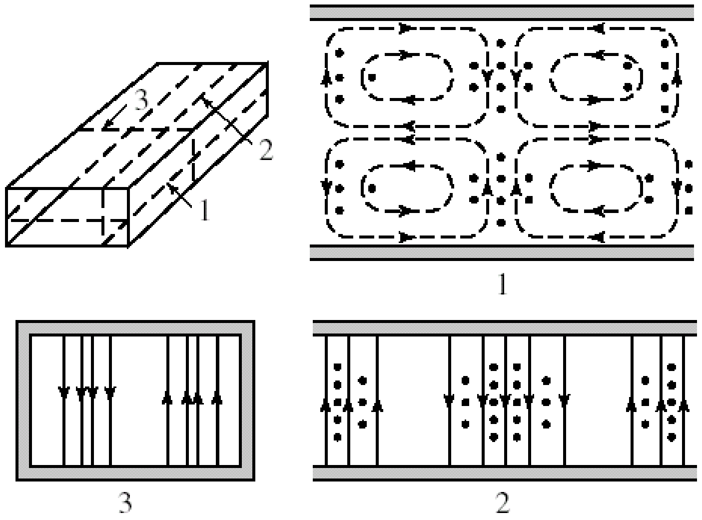
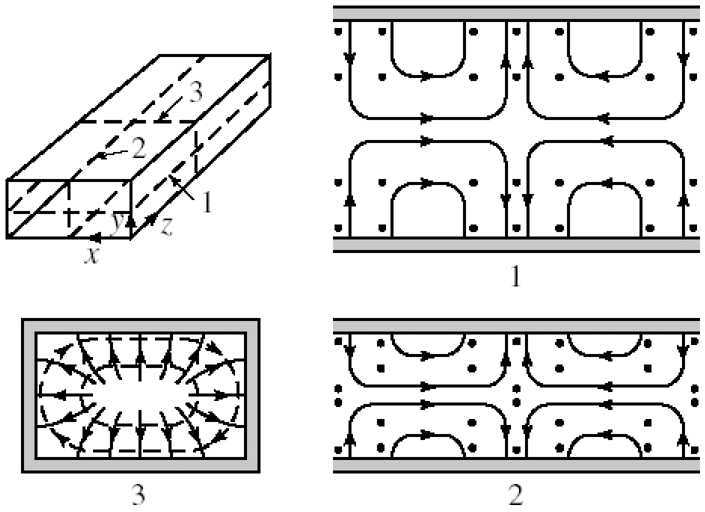
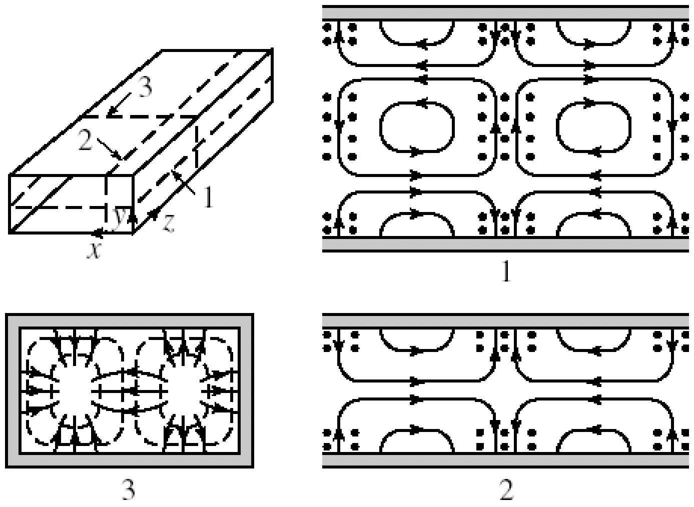
Dominant and Degenerate Modes

- Then the characteristic impedance for this mode is $$\eta_{nmTE} = Z_{nmTE} = Z_{nmH} = -\frac{E_y}{H_x}$$

$$\Rightarrow \eta_{10TE} = Z_{10TE} = Z_{10H} = Z_0 \frac{k_0}{\beta_{10}}$$

- The guided wavelength is also modeled as $$\lambda_g = \frac{2\pi}{\beta_g} \text{ or} \lambda_{10} = \frac{2\pi}{\beta_{10}} = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_{c10}}\right)^2}} = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}}$$

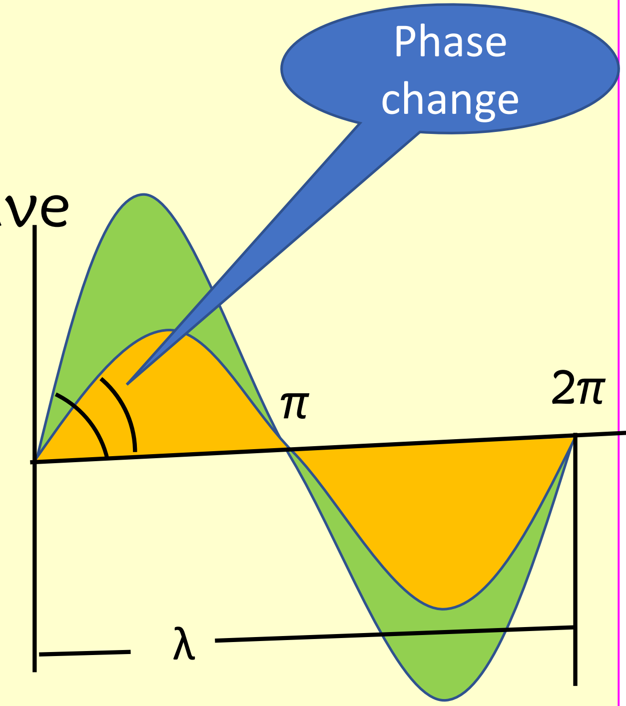
- **Degenerate** (or Evanescent) **modes** are modes below the cutoff frequency.
- They cannot propagate down the waveguide for any distance, dying away exponentially.
- However, they are important in the functioning of filter components such as irises and posts, because energy is stored in the evanescent wave fields
- At lower frequencies the waveguide needs to be impractically large in order to keep the cutoff frequency below the operational frequency.

TE_{10}	TE_{11}	TE_{21}
		
TE_{20}	TM_{11}	TM_{21}
		

Mode characteristics: Phase Velocity Vp and Group Velocity Vg

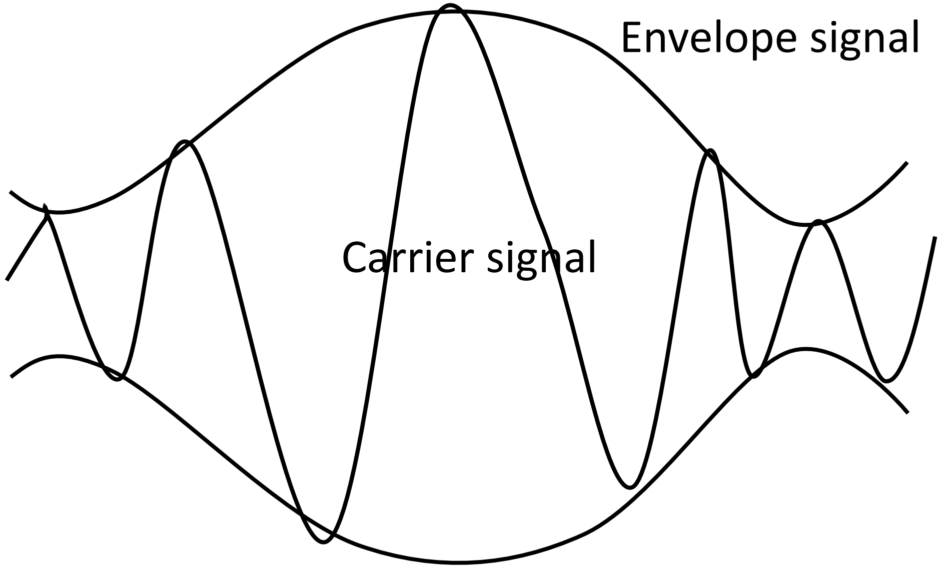
- **Phase Velocity (Vp)** is the rate at which the wave changes its phase in order to undergo a phase shift of 2π radians.
- It can be understood as the change in velocity of the wave components of a sine wave, when modulated.
- The equation for the Phase velocity can be derived as follows;
- We know V=λ/T; Where, λ = wavelength and T = time
- It can be V=λ/T=λ.f (Since f=1/T)
- If we multiply the numerator and denominator by 2π then, we have
- $V=\lambda f=2\pi\lambda f/2\pi$
- We know that $\omega=2\pi f$ and $\beta=2\pi/\lambda$
- The above equation can be written as $V=2\pi f/(2\pi/\lambda)=\omega/\beta$
- Hence, the equation for Phase velocity is represented as $V_p=\omega/\beta$
- Or it can be

$$\upsilon_p=\frac{c}{\sqrt{1-\left(f_c/f_0\right)^2}}=\frac{c}{\sqrt{1-\left(\lambda_0/\lambda_c\right)^2}}$$



Mode characteristics: Phase Velocity Vp and Group Velocity Vg

- **Group Velocity** Vg can be defined as the rate at which the wave propagates through the waveguide.
- This can be understood as the rate at which a modulated envelope travels compared to the carrier alone.
- This modulated wave travels through the waveguide.
- The equation of Group Velocity is represented as Vg=dω/dβ
- The velocity of modulated envelope is usually slower than the carrier signal.



Vg=dω/dβ **or**

$$\upsilon_g = c\sqrt{1-\left(\frac{f_c}{f_0}\right)^2} = c\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

Power Transmitted in RWGs

•Power transmitted in a rectangular waveguide is given by

$$P = |E| \, X \, |H|$$

$P_x =$ Not required

Power flow in x direction

$P_y =$ Not required

Power flow in y direction

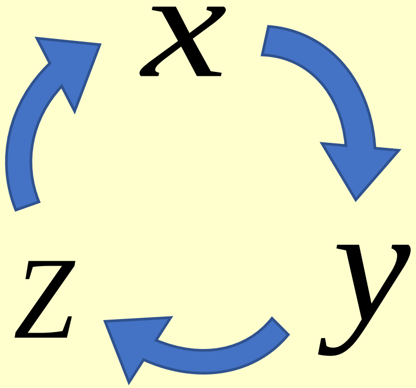
Power directed in +ve Z direction

$$P_z = |E_x| \, X \, |H_y|$$

or

Power directed in –ve Z direction

$$P_z = -\left(|E_y| \, X \, |H_x|\right)$$



•As mentioned, there are three possible powers with the availabilities of E and H wrt its coordinates as shown.

•Out of the list, the component of interest is the **one which is moving in the direction of Z**(from Z=0 to Z= infinity.)

- Then the power propagated along Z is given by $P_{nmz} = E_x . H_y$ $\quad P_{tz} = \oint E_x . H_y \, ds$

- And $(P_{nmz})_{total} = \int_{x=0}^{a} \int_{y=0}^{b} E_x . H_y \, dy.dx$ or $(P_{nmz})_{total} = \int_{x=0}^{a} \int_{y=0}^{b} \left(\frac{E_x}{\sqrt{2}} . \frac{H_y}{\sqrt{2}} \right) dy.dx$

$$\Rightarrow (P_{nmz})_{total} = \frac{1}{2} \int_{x=0}^{a} \int_{y=0}^{b} \left(E_x . H_y \right) dy.dx$$

- But the relationship between the characteristic impedance and E-M fields is

$$\eta_{nm} = \frac{E_x}{H_y} \text{ or } E_x = \eta_{nm} H_y \text{ or } H_y = \frac{E_x}{\eta_{nm}}$$

- So $(P_{nmz})_{total} = \frac{1}{2} \int_{x=0}^{a} \int_{y=0}^{b} \left(\eta_{nm} H_y . H_y \right) dy.dx$ or $(P_{nmz})_{total} = \frac{1}{2} \int_{x=0}^{a} \int_{y=0}^{b} \left(E_x . \frac{E_x}{\eta_{nm}} \right) dy.dx$

- Re arranging the terms will results;

$$(P_{nmz})_{total} = \frac{\eta_{nm}}{2} \int_{x=0}^{a} \int_{y=0}^{b} \left| H_y \right|^2 dy.dx \text{ or } (P_{nmz})_{total} = \frac{1}{2\eta_{nm}} \int_{x=0}^{a} \int_{y=0}^{b} \left| E_x \right|^2 dy.dx$$

- So the total power is $$(P_{nmz})_{total} = \frac{1}{2\eta_{nm}} \int_{x=0}^{a} \int_{y=0}^{b} |E_x|^2 \; dy.dx$$

$$(P_{nmz})_{total} = \frac{1}{2\eta_{nm}} \int_{x=0}^{a} \int_{y=0}^{b} \left| \frac{\omega\mu}{k_c^2} A_{nm} \frac{m\pi}{b} \mathrm{Cos}(\frac{n\pi}{a} x) \mathrm{Sin}(\frac{m\pi}{b} y) \right|^2 dy.dx$$

$$\Rightarrow (P_{nmz})_{total} = \frac{1}{2\eta_{nm}} \left(A_{nm} . \frac{\omega\mu}{k_c^2} . \frac{m\pi}{b} \right)^2 \int_{x=0}^{a} \int_{y=0}^{b} \mathrm{Cos}^2(\frac{n\pi}{a} x) \mathrm{Sin}^2(\frac{m\pi}{b} y) dydx$$

from the fundamental knowledge

$$\int_{x=0}^{a} \int_{y=0}^{b} \mathrm{Cos}^2(\frac{n\pi}{a} x) \mathrm{Sin}^2(\frac{m\pi}{b} y) dydx = \begin{cases} \frac{ab}{4}; & \text{when n} \neq 0 \text{ and m} \neq 0 \\ \frac{ab}{2}; & \text{when n} = 0 \text{ and m} = 0 \end{cases}$$

- Hence the final power is $$\therefore (P_{nmz})_{total} = \frac{1}{2\eta_{nm}} \left(A_{nm} . \frac{\omega\mu}{k_c^2} . \frac{m\pi}{b} \right)^2 . \frac{ab}{4}$$

Power Losses in Rectangular Guide

- For waveguides with conducting walls, the transmission (or ohmic) losses are of two types:
- (a) loss due to dielectric medium α_d
- (b) loss due to conductor walls α_c
- In dielectric waveguides, losses are due to absorption and scattering by imperfections
- If the operating frequency is less than the cut-off frequency, then the attenuation is more and the mode turns into non-propagating mode or degenerate mode
- The transmission losses can be quantified by replacing the propagation constant β by its complex-valued version as $\alpha + j\beta$ in the general expressions.
- As usual, the power loss can be defined by using Poynting Vector(P=EH); where in E and H are defined in terms of attenuation (α_c),as shown below.

$$E_z = |E_{0z}| e^{-\gamma_g z} = |E_{0z}| e^{-(\alpha_g + j\beta_g)z} = |E_{0z}| e^{-\alpha_g z} e^{-j\beta z}$$

$$\Rightarrow |E_z| = |E_{0z}| e^{-\alpha_g z} \text{ and}$$

$$\text{llly } |H_z| = |H_{0z}| e^{-\alpha_g z}$$

Power Transmission and Power Losses in Rectangular Guide

- Let elaborate the sub components causing for the losses in the propagation
- The first one is, attenuation is a function of dielectric constant(μ and ε) and is given by

$$\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

- The second one is, attenuation is a function of operating frequency(fc) and is given by

$$\alpha_g = \frac{\sigma\eta}{2\sqrt{1-\frac{f_c}{f_0}}} \text{;for TE mode} \qquad \alpha_g = \frac{\sigma\eta}{2}\sqrt{1-\frac{f_c}{f_0}}\text{; for TM mode}$$

- It is interesting to note that, for a low loss guide, the time average power flow decreases proportionally to $e^{-2\alpha_g z}$ hence,

$$P_r = P_{tz} e^{-2\alpha z} \quad \text{where } \alpha = \alpha_d + \alpha_c$$

- Power loss is also defined as power per unit length, as given in below equation

$$P_{loss} = \frac{P_{tz}}{z}$$ and the attenuation can also be given as $$\alpha_t = \frac{P_{loss}}{2P_{tz}\, z}$$

Related Problems: Example1

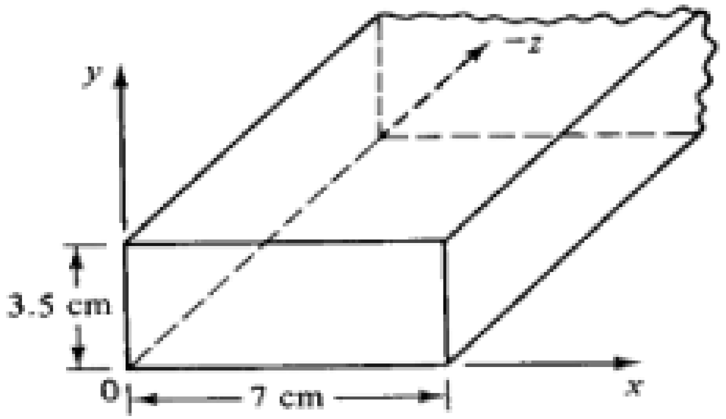
- An air-filled rectangular waveguide of inside dimensions 7 x 3.5 cm operates in the dominant TE10 mode operating at a frequency of 3.5GHz as shown in Fig. Then find
- Cut off frequency,
- Guided wavelength,
- Group velocity
- Phase Velocity
- Phase constant
- Cut off wavenumber

$$\lambda_0 = \frac{c}{f_0}$$

$$1.\ f_{c10} = \frac{c}{2a}$$

$$\upsilon_g = c\sqrt{1-\left(f_c/f_0\right)^2} = c\sqrt{1-\left(\lambda_0/\lambda_c\right)^2}$$

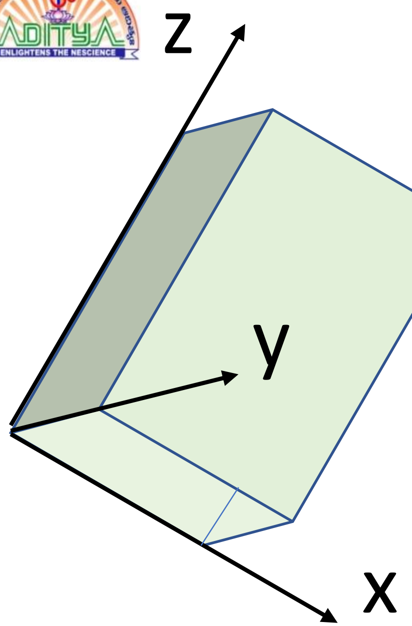
$$\upsilon_p = \frac{c}{\sqrt{1-\left(f_c/f_0\right)^2}} = \frac{c}{\sqrt{1-\left(\lambda_0/\lambda_c\right)^2}}$$



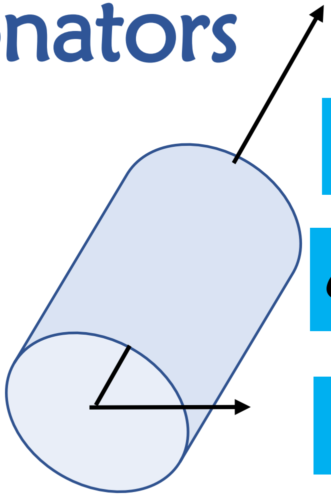
$$\lambda_g = \frac{2\pi}{\beta_g} \text{ or } \lambda_{10} = \frac{2\pi}{\beta_{10}} = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_{c10}}\right)^2}} = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}}$$

$$\Rightarrow \beta_{10}^2 = k_0^2 - k_{c\,10}^2 \text{ or } \beta_{10} = \left(k_0^2 - \left(\frac{\pi}{a}\right)^2\right)^{\frac{1}{2}}$$

Cavity Resonators



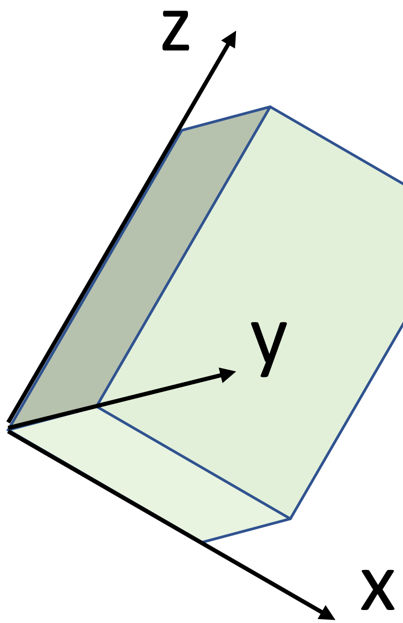
X=0-to-a,
Y=0-to-b, &
Z=0-to-α
And c > a > b



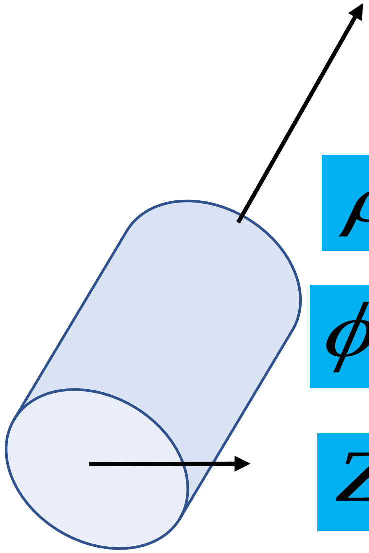
$\rho = 0\, to\, a$

$\phi = 0\ \ to\, 2\pi$

$Z = 0\, to\, \alpha$



X=0-to-a,
Y=0-to-b, &
Z=0-to-c
And c > a > b



$\rho = 0\, to\, a$

$\phi = 0\ \ to\, 2\pi$

$Z = 0\, to\, d$

- If we close off two ends of a waveguide with metallic walls we have a cavity resonator
- In this case, the wave propagating in the z- direction will bounce off the two walls resulting in a standing wave in the z- direction
- Resonator is a tuned circuit which resonates at a particular frequency at which the energy stored in the electric field is equal to the energy stored in the magnetic field.
- Resonant frequency of microwave resonator is the frequency at which the energy in the resonator attains maximum value. i.e., twice the electric energy or magnetic energy.
- At low frequencies up to VHF (300 MHz), the resonator is made up of the reactive elements or the lumped elements like the capacitance and the inductance.
- Transmission line resonator can be built using distributed elements like sections of coaxial lines(either opened or shorted) at the end sections thus confining the electromagnetic energy within the section and acts as the resonant circuit having a natural resonant frequency.

•At very **high frequencies** transmission **line resonator** does **not give** very **high quality** factor Q due to skin effect and radiation loss.

•So, transmission line resonator is not used as microwave resonator.

•The performance parameters of microwave resonator are:

(i) Resonant frequency

(ii) Quality factor

(iii) Input impedance

Quality Factor of a Resonator:

- The quality factor Q is a measure of frequency selectivity of the resonator.
- The quality factor Q defined as

$$Q = 2\pi \times \text{Maximum energy stored / Energy dissipated per cycle} = W / P$$

Where,

a. W is the maximum stored energy

b. P is the average power loss

Rectangular Cavity Resonators Field Exp.s for TE & TM mode

• In the case of TE and TM mode of Rectangular cavity resonator, the Z-component is perpendicular to X- and Y- and hence the field expressions are given by

$$H_z = A_{nm}\,\mathrm{Cos}\left(\frac{n\pi}{a}x\right)\mathrm{Cos}\left(\frac{m\pi}{b}y\right).\mathrm{Sin}\left(\frac{l\pi}{c}z\right);\ \text{for TE mode}$$

$$E_z = A_{nm}\,\mathrm{Sin}\left(\frac{n\pi}{a}x\right)\mathrm{Sin}\left(\frac{m\pi}{b}y\right).\mathrm{Cos}\left(\frac{l\pi}{c}z\right);\ \text{for TM mode}$$

Cut off wave number k_c is defined as

$$k_c = \sqrt{k_x^2 + k_y^2 + k_z^2} = \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 + \left(\frac{l\pi}{c}\right)^2}$$

Resonant frequency f_r is defined as

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 + \left(\frac{l\pi}{c}\right)^2}$$

Power oscillating in rectangular cavity resonator is

$$\left(P_{nmz}\right)_{total} = \int_{z=0}^{c}\int_{x=0}^{a}\int_{y=0}^{b}\left(\frac{E_x}{\sqrt{2}}.\frac{H_y}{\sqrt{2}}\right)dy.dx.dz$$

Circular Cavity Resonators Field Exp.s for TE & TM mode

- A circular cavity resonator is a closed microwave structure useful in signal generators.
- If we close off two ends of a waveguide with metallic walls we have a circular cavity resonator
- In the case of TE and TM mode of circular cavity resonator, the Z-component is perpendicular to radial- and circumferential components and hence the field expressions are given by

$$H_z = C_n J_n(k_c\rho).C_n^1 \ \mathrm{Cos}[n\phi]^1 \ \mathrm{Sin}(\frac{l\pi}{c}z);\text{for TE mode}$$

$$E_z = C_0 J_n(k_c\rho).\ \mathrm{Cos}[n\phi]^1 .\mathrm{Sin}(\frac{l\pi}{c}z);\ \text{for TM mode}$$

Power oscillating in circular cavity resonator is

$$(P_{nmz})_{total} = \int_{z=0}^{c}\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\left(\frac{E_\rho}{\sqrt{2}}.\frac{H_\phi}{\sqrt{2}}\right).d\phi.d\rho.dz$$

Cut off wave number k_c is defined as

$$k_c = \sqrt{k_\rho^{\ 2} + k_\phi^{\ 2}} = \sqrt{\left(\frac{P_{nm}^{\ 1}}{a}\right)^2 + \left(\frac{l\pi}{c}\right)^2}$$

Resonant frequency f_r is defined as

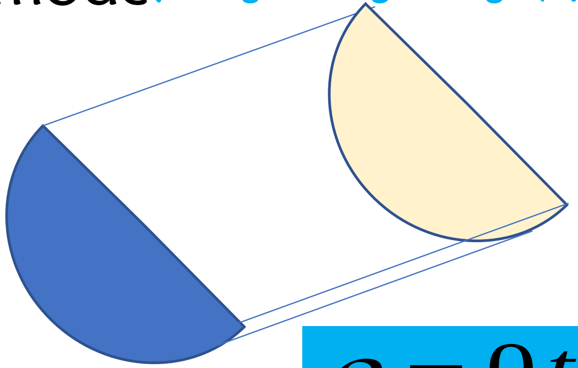
$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{P_{nm}^{\ 1}}{a}\right)^2 + \left(\frac{l\pi}{c}\right)^2}\ ;\ for TE$$

Resonant frequency f_r is defined as

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{l\pi}{c}\right)^2}\ ;\ for TM$$

Semi-Circular Cavity Resonators Field Exp.s for TE & TM mode

- A circular cavity resonator whose circumferential variation is limited to half.
- In the **Semi-circular cavity** resonator, the field components are **as same as** normal cavity resonators
- But the difference is in its total power calculation, which is taking the half circle variations instead of full circle variations



$\rho = 0 \, to \, a$

$\phi = 0 \, to \, \pi$

$Z = 0 \, to \, d$

$$H_z = C_n J_n(k_c\rho).C_n^1 \, Cos[n\phi]^1 \, Sin(\frac{l\pi}{c}z); \text{for TE mode}$$

$$E_z = C_0 J_n(k_c\rho). \, Cos[n\phi]^1 .Sin(\frac{l\pi}{c}z); \text{ for TM mode}$$

Power oscillating in circular cavity resonator is

$$(P_{nmz})_{total} = \int_{z=0}^{c}\int_{\rho=0}^{a}\int_{\phi=0}^{\pi}\left(\frac{E_\rho}{\sqrt{2}}.\frac{H_\phi}{\sqrt{2}}\right).d\phi.d\rho.dz$$

Cut off wave number k_c is defined as

$$k_c = \sqrt{k_\rho^{\,2} + k_\phi^{\,2}} = \sqrt{\left(\frac{P_{nm}^{\,1}}{a}\right)^2 + \left(\frac{l\pi}{c}\right)^2}$$

Resonant frequency f_r is defined as

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{P_{nm}^{\,1}}{a}\right)^2 + \left(\frac{l\pi}{c}\right)^2} \; ; forTE$$

Resonant frequency f_r is defined as

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{l\pi}{c}\right)^2} \; ; forTM$$

Quality factor of a Cavity Resonator

- Quality Factor of a Resonator:
- The quality factor Q is a measure of frequency selectivity of the resonator.
- The quality factor Q defined as

$$Q = 2\pi X\left[\frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}\right] \quad \text{or } Q = 2\pi X\left[\frac{\text{Maximum energy stored}}{\text{Energy dissipated}}\right] = \frac{wW}{P}$$

- Where,

  W is the maximum stored energy

  P is the average power loss

$$\Rightarrow Q = \frac{wW}{P} = \frac{w\frac{\mu}{2}\int_v |H|^2\, dv}{\frac{R_s}{2}\int_s |H_t|^2\, da} = \frac{w\mu\int_v |H|^2\, dv}{R_s\int_s |H_t|^2\, da} \qquad \text{where } |H|^2 = |H_t|^2 + |H_n|^2$$

$$\Rightarrow Q = \frac{w\mu}{2R_s};\ \text{since } H_t = 2H \text{ at the resonator walls}$$

Quality factor of a loaded and unloaded Cavity Resonator

•An unloaded resonator can be represented by a series or parallel resonant circuit . The resonant frequency fr and the unloaded Qo of a cavity resonator is given by

$$f_r = \frac{1}{2\pi\sqrt{LC}} \text{ and } Q_0 = \frac{\omega_0 L}{R}$$

•The loaded Ql of a cavity resonator is given by $Q_l = \frac{\omega_0 L}{R + N^2 Z_g}$

Where N is primary turns of the load transformer

Zg is the internal impedance of the source

•The relation between loaded and unloaded quality factors is given by

$$Q_l = \frac{\omega_0 L}{R\left(1 + \frac{N^2 Z_g}{R}\right)} = \frac{\omega_0 L}{R(1+K)}; \text{ with K as coupling coefficient}$$

$$\therefore Q_l = \frac{Q_0}{(1+K)};$$

Types of Coupling with coupling coefficient, K

- There are three types of coupling coefficients
- **Critical coupling(K=1):**
- Then the loaded Quality factor is $Q_l = \frac{Q_0}{2};$
- **Over coupling(K>1):**
- In this case, the cavity terminals are at a voltage maximum in the input line at resonance. The normal impedance is the standing wave ratio ρ then the loaded Quality factor is $$Q_l = \frac{Q_0}{(1+\rho)};$$
- **Under coupling(K<1):**
- In this case, the cavity terminals are at a voltage minimum in the input line at resonance, and the input impedance is the the reciprocal of standing wave ratio ρ then the loaded Quality factor is

$$\therefore Q_l = \frac{Q_0}{\left(1+\frac{1}{\rho}\right)} = \frac{Q_0}{\left(\frac{\rho+1}{\rho}\right)}; \Rightarrow Q_l = Q_0\frac{\rho}{\rho+1};$$